



Stochastic Dynamic Programming for Resource Management

AMSI Workshop, July 2009

Energy Edge

- Energy Edge business services:
 - Assist clients with
 - Trading,
 - Commercial,
 - Decision support,
 - Commodity management.
 - Optimise tradeoff between profitability and risk
 - Optimisation subject to constraints
 - Real constraints: limited resources
 - Imposed constraints: Board risk appetite

Overview

- Three-part workshop
 - Part 1: Stochastic models
 - Describing and modelling the uncertainty
 - Part 2: Connection between electricity and water
 - Supply and demand influences
 - Part 3: Stochastic dynamic programming
 - Optimal management of a limited resource

Stochastic Dynamic Programming

- An Operations Research device
- Developed in 1950s
 - “An optimal policy has the property that whatever the initial state and initial decisions are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision”
 - Bellman principal of optimality
- Ideal for allocating **limited resources** to achieve a single **objective** and to calculate **value** and **strategy** under **uncertainty**.

This workshop

- Reformulate the electricity and water resource problem ready for SDE
 - Previously in SDE form
- Introduce Bellman principle of optimality
- Formulate an electricity problems as SDP
- Solve a problem in electricity.

Stochastic processes

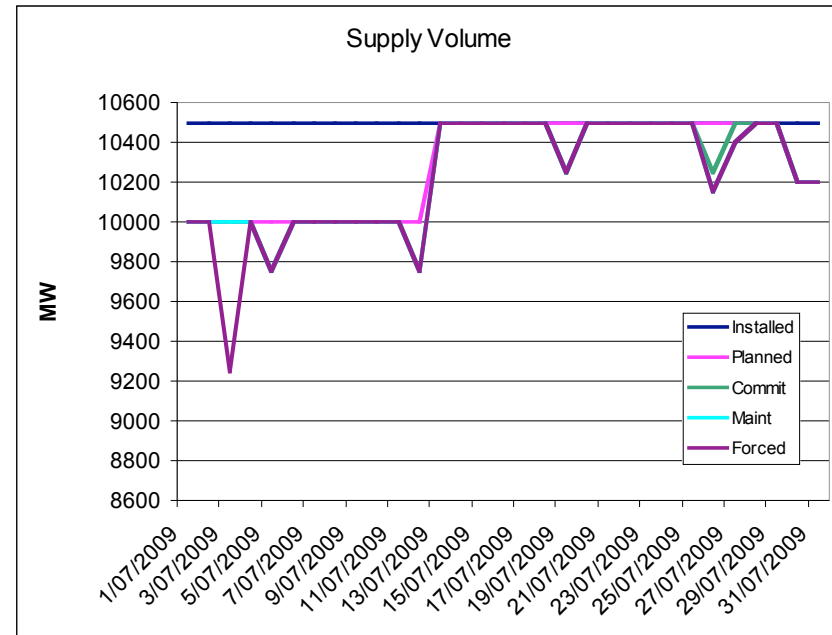
- Workshop 1:
 - Demand modeled as a diffusion process
- There are other stochastic modeling tools
 - Poisson processes
 - Markov chains

Poisson process

- Diffusion processes are useful for modelling *continuous* variables
- Poisson process for modelling *shock* events
 - Plant outages
 - Electricity price spikes

Modelling supply availability

- Power station availability contains deterministic and random components
 - Installed capacity **less**
 - Planned outages **less**
 - Commitment decision **less**
 - Plant maintenance **less**
 - Plant forced outages



Typical approaches

- Independent experiments
 - Mean time to failure
 - Mean time to repair
- Poisson process
 - Exponential distribution
 - Weibull distribution

Availability model implementation

Failure time is exponential distribution: PDF

$$f(x) = \lambda e^{-\lambda x}$$

1. Model failure times directly: CDF

$$F(x) = \int_0^x f(s) ds = 1 - e^{-\lambda x}$$

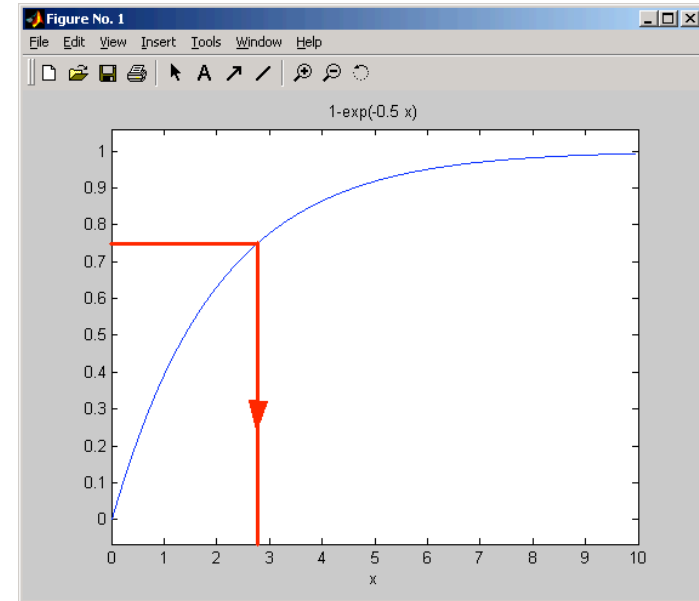
Let $R \sim U(0,1)$ then

$$X = F^{-1}(R) = -\log(1-R)/\lambda$$

2. Model failure times directly:

$\Pr(\text{failure in } [0, 0+dt])$

$$= 1 - e^{-\lambda dt} \approx \lambda dt + O(dt^2)$$

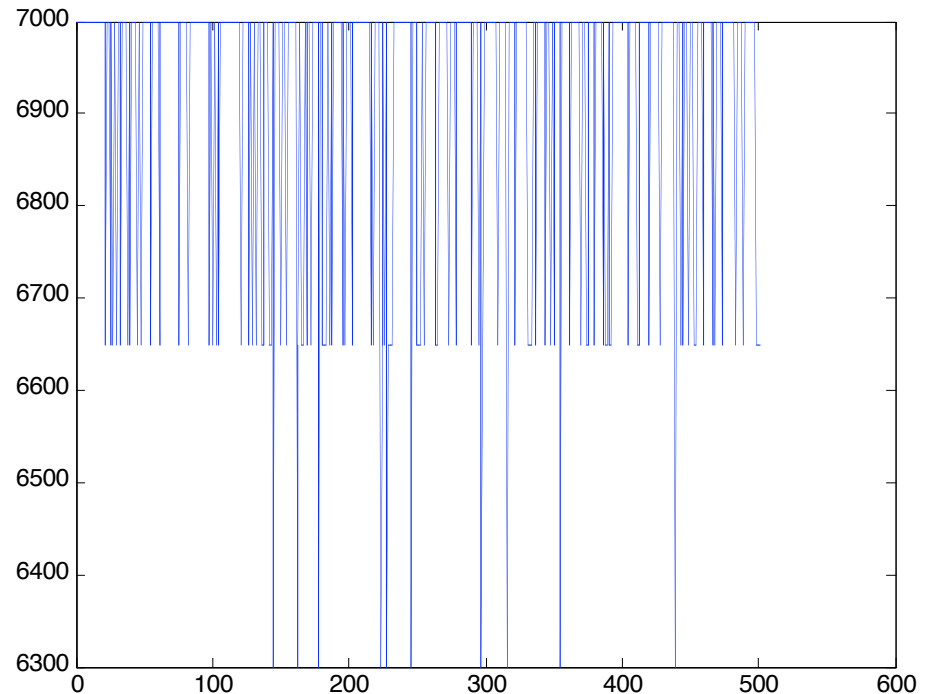


Time-dependent hazard

- Parameter λ is the hazard or intensity
- Larger λ , larger chance of failure
- May be time varying
 - Not an exponential distribution for failure times
 - Recover other distributions from the Poisson process
- Interestingly
 - Power stations are more reliable the longer they are ON

Aggregate effect

- The effect of many forced outages is shown here.
- 20 generating units, 350 MW each
- 1% chance of failure in any timestep
- 500 timesteps
- 30 simulations



Model with Mean Reversion

- “Mean reverting” SDE a common approach in many commodity markets
- Appeal to central limit theorem
- Even more marked effect with more units and partial outages

Markov chains

- Given a state space $\{X_1, \dots, X_n\}$
- Given a probability transition matrix P
- The state makes transitions from X_i to X_j with probability P_{ij} over time step dt .
- The transition is not dependent on the history, only the present state

Applications

- Outage state modelling
- Credit modelling
- Demand modelling

Outage state

Over the next timestep, the chance of going from **100% avail** to **failed** is 2%.

		To state		
		100% avail	50% avail	Failed
From State	100% avail	0.95	0.03	0.02
	50% avail	0.4	0.3	0.3
	Failed	0.05	0.65	0.3

Queries on outage states

- Probability of ending up in a failed state, starting from a 100% available state after 100 steps?
- Answer from: P^{100}

		To state		
		100% avail	50% avail	Failed
From State	100% avail	0.8381	0.0966	0.0653
	50% avail	0.8381	0.0966	0.0653
	Failed	0.8381	0.0966	0.0653

Credit risk analysis

- Credit migration model
- Introduce an absorbing state: failed = “default”
- Probability of eventual default? 1.

		To state		
		AAA	BBB	Failed
From State	AAA	0.95	0.03	0.02
	BBB	0.4	0.3	0.3
	Failed	0	0	1

Financial risk from credit

- I am only concerned with credit default if *they owe me* money.
- Two processes:
 - Market prices – how much value do they owe me.
 - Credit processes – how likely will they fall into default when they owe me.
 - Question: Is there a dependence?
- Mechanisms exist to manage the risk.

Applications to SDE

- SDE models:

$$dS = a(S,t) dt + b(S,t) dW$$

Previously W was simulated as a normal random variable.

Now set up a discrete grid of allowable S states.

Discretise dS into permissible buckets

- Advantage

- I now have a known and finite set of states.
- Useful for management purposes.

- Alternatives:

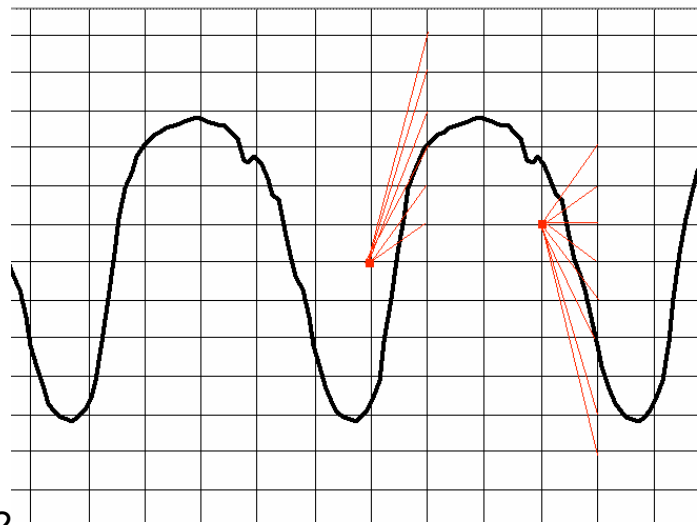
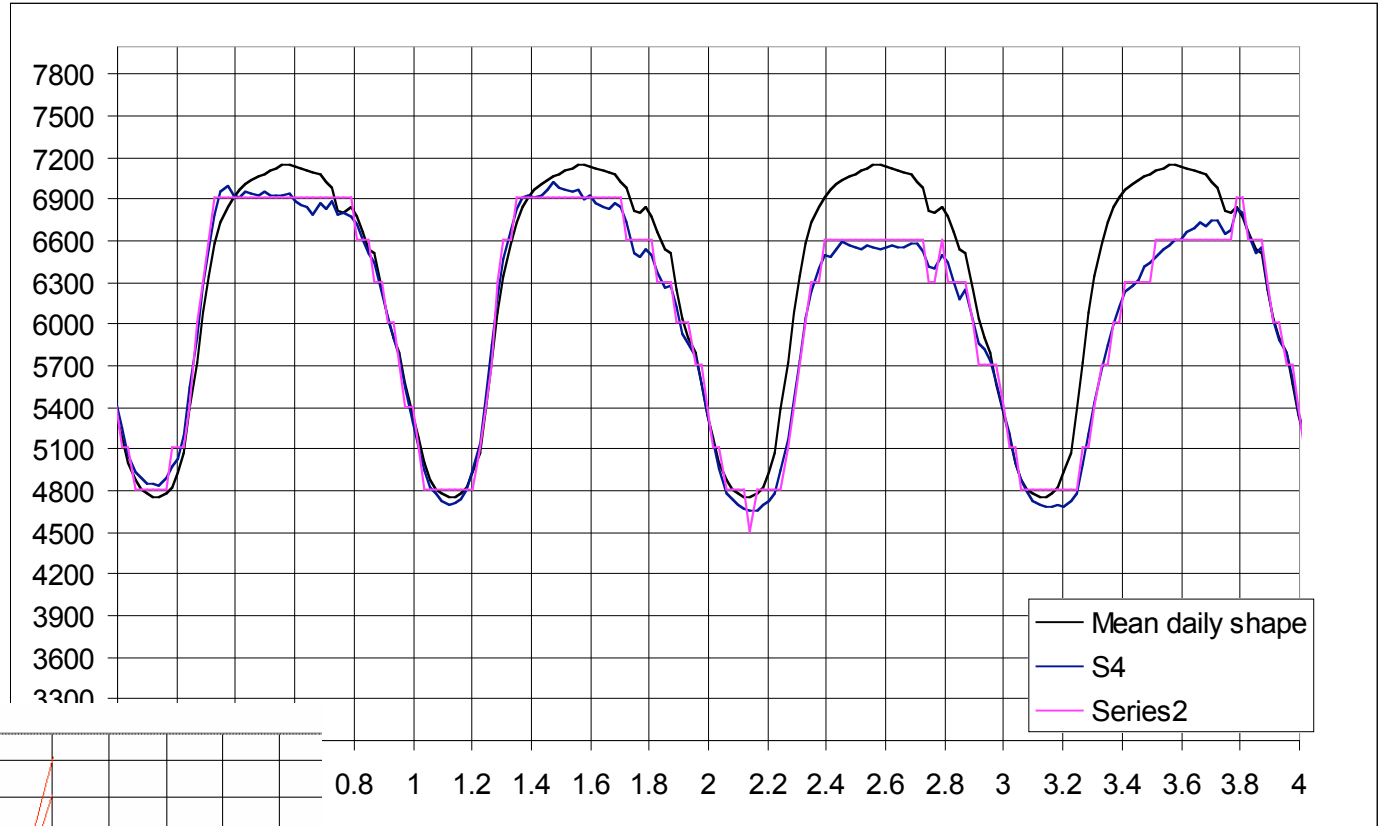
- I can customise probability transition matrix to discrete grid
- I can customise discrete grid to PTM

$$dS = a(S,t) dt + b(S,t) dW$$
$$dW \sim N(0, dt)$$

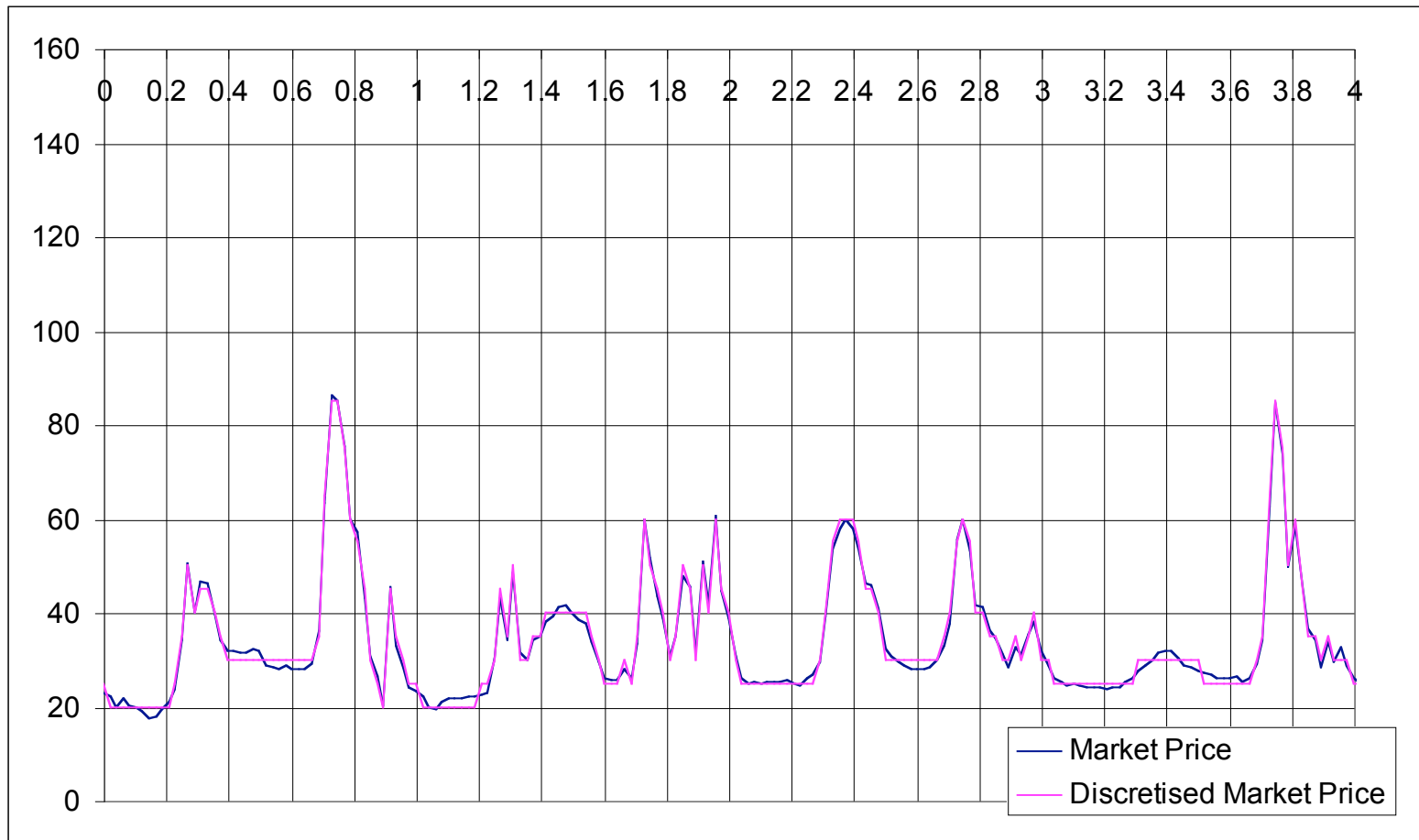
Now

$$S(t) = S(t) + [a(S,t) dt + b(S,t) dW]$$

Where [.] maps to an appropriate grid point



Similarly with price

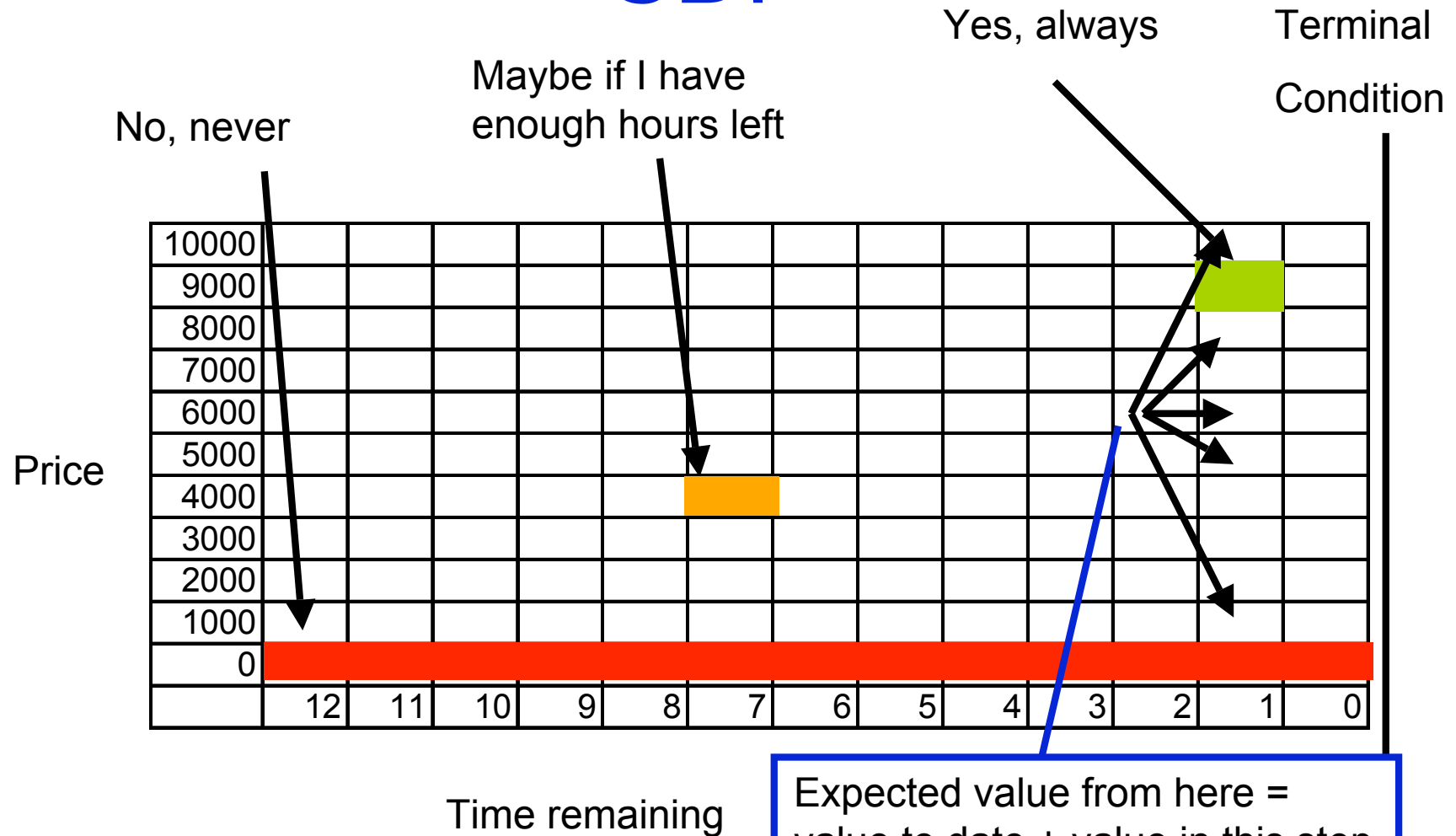


Our Problem

- I have a generator which only has enough water to generate for 100 hours per year.
- I should generate when the price is high.
- How high is high enough?

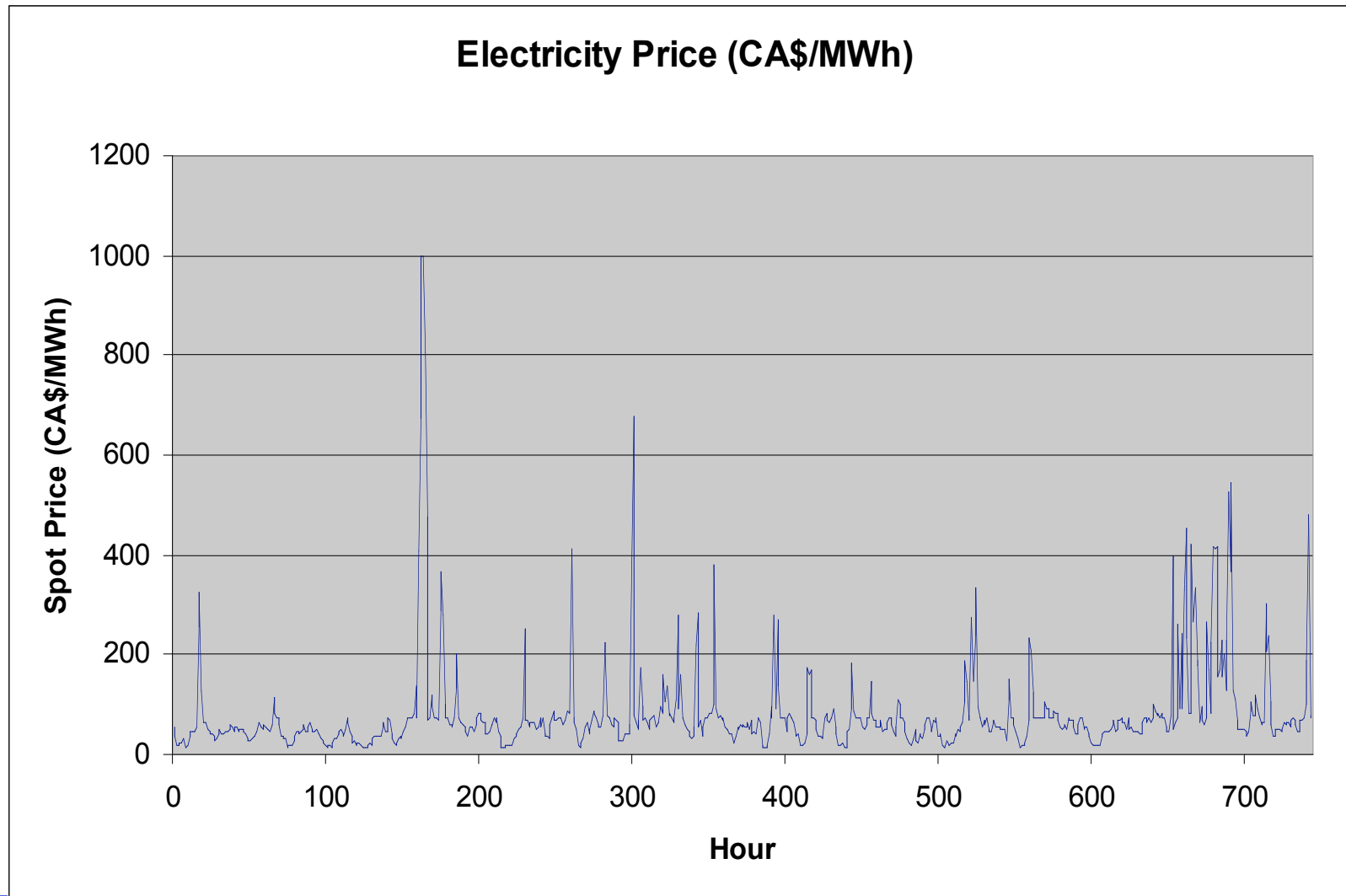
- I have a process which can close down (curtail) for 100 hours per year (eg water pumping process)
- I should shut down when the price is high.
- How high is high enough?

SDP



Expected value from here =
 value to date + value in this step
 + value from future steps under
 optimal strategy

Calculated on Alberta market



The Model

- Electricity spot prices obey a Markov process.
- The control strategy is undertaken based on the *state* of the system, where the state is defined by:
 - The number of hours into the year
 - The prevailing electricity spot price,
 - Whether the contract is currently in a state of curtailment or not curtailed,
 - The amount of curtailed hours used already and the amount of curtailable hours available for rest of year.
- There is no forward-looking capability apart from the current spot price, and a known statistical behaviour from that spot price.
- An hourly resolution for the modelling is assumed.
- Neglect the time value of money in the analysis.

- The fair value of the curtailment optionality is the expected payoff for the contract under an optimal strategy of curtailment less the expected payoff for the contract under the alternative of never curtailing.
- The optimal strategy is defined as the set of state-dependent decisions to commence or conclude curtailment, which maximizes the expected payoff of the contract.
- The primary modeling methodology is based on a *stochastic dynamic program*. The solution of the dynamic program yields both the optimal exercise *strategy* and the *value* of the curtailment.

Stochastic Dynamic Program Algorithm

Let

dt = time step in hours (1 hour for the hourly Alberta market)

K = fixed price in \$/MWh for contract during periods when not curtailed

C = contract volume in MW

Y = total length of the curtailment period (one year = 8760 time steps)

t = time steps into the current year

H = total inventory of curtailment hours under contract (100 hours)

h = total number of curtailment units used to date

M = indicator variable

= 0 if system is in an uncurtailed state (normal operating)

= ∞ if system is currently in a curtailed state

= m if system is uncurtailed, but curtailment has been called and will start in m timesteps.

= $-m$ if system is curtailed, but conclusion to curtailment has been called and will end in m timesteps.

Stochastic Dynamic Program Algorithm

S = spot price state (discretised set of possible spot prices)

J = number of spot prices in discretisation (S_1, \dots, S_J)

D = delay of time steps from calling curtailment until it commences (two hours).

E = delay of time steps from calling end to curtailment until it concludes (zero hours)

$P(S_1, S_2)$ = probability that the spot price will make a transition from price state S_1 to price S_2 over time step dt at time t .

Define the state of the system by the tuple: $X = (t, S, h, M)$ and let the value remaining in the curtailment contract be $V(t, S, h, M)$. We present the value function at state X dependent on the value functions at the possible next state X' . Over that time step, we assume that the optimal curtailment control is applied to maximize the total value. The possible controls available are to do nothing, to curtail from an uncurtailed state, or to uncurtail from a curtailed state.

Stochastic Dynamic Program Algorithm

The terminal condition states that when the end of the year is reached ($t = T$), there is no more value to be extracted from curtailment because time has 'run out':

$$V(T, S, h, M) = 0$$

for all S, h, M

The boundary condition on remaining resources H dictates that when the total hours available for curtailment have been used ($H = 0$), then no more value can be extracted from the contract:

$$V(t, S, 0, M) = 0$$

for all t, S, M

Stochastic Dynamic Program Algorithm

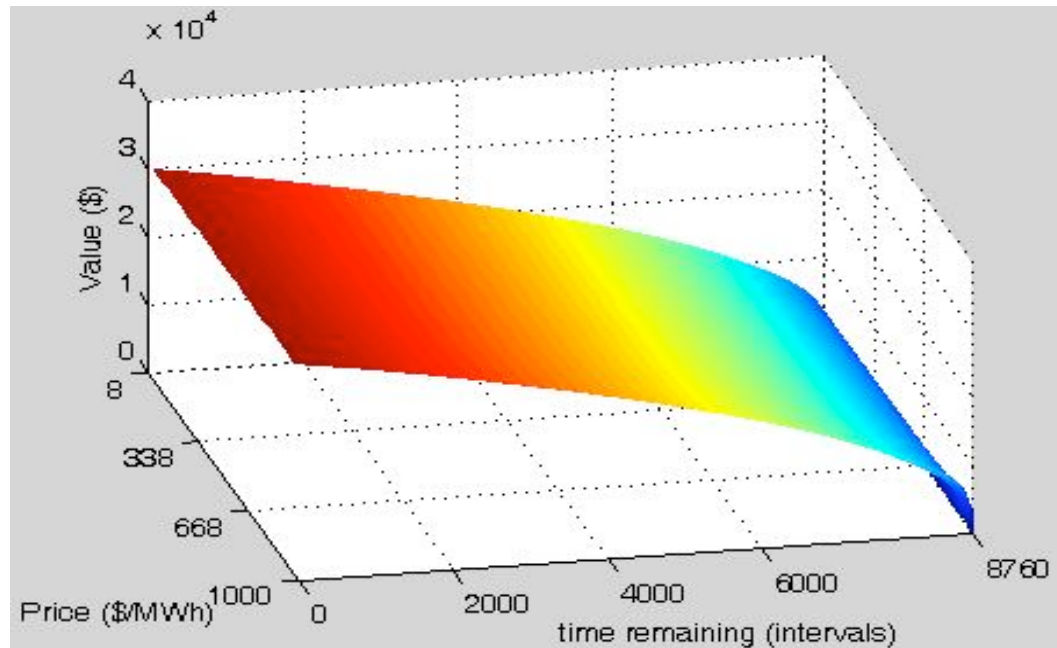
We have the following 4 regimes:

- system is not curtailed ($m = 0$),
- curtailment has been called but has not yet started ($m > 0$),
- curtailment is in place or ($m = \infty$)
- curtailment is in place but conclusion has been called ($m < 0$)

The dynamic program is implemented under each regime. The recursion relation states that the value in the next time step is the value which is extracted from any curtailment in the current time step, plus the expected value of the contract after the next time step, conditioned on the uncertain movements in spot prices.

Results and Discussion

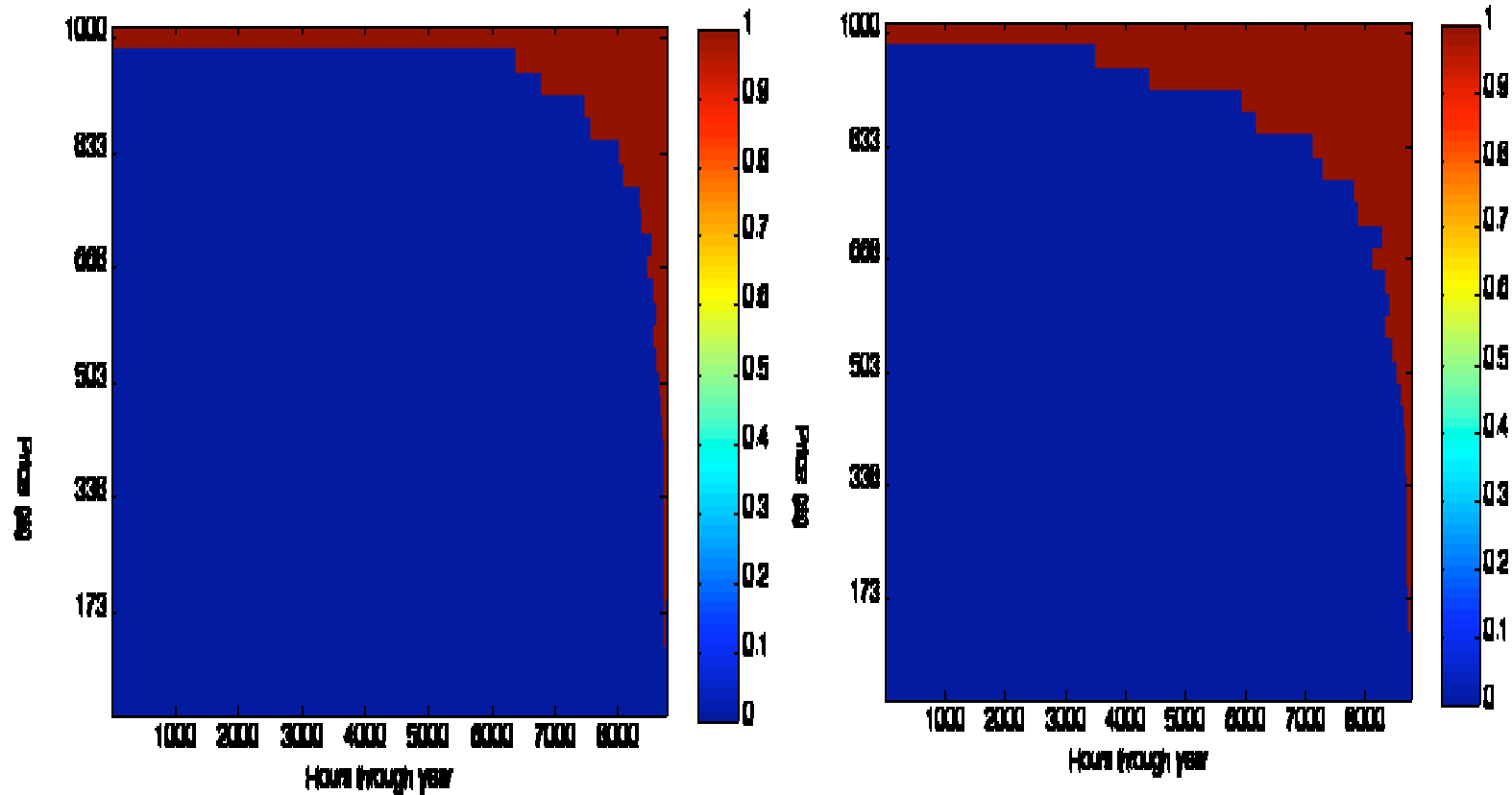
The SDP is applied on a grid of $J = 31$ spot prices uniformly distributed in a log scale between \$8/MWh and \$1,000/MWh. We have calibrated that the fixed price is $K = \$61/\text{MWh}$. The calculation is performed on a notional volume $C = 1 \text{ MW}$.



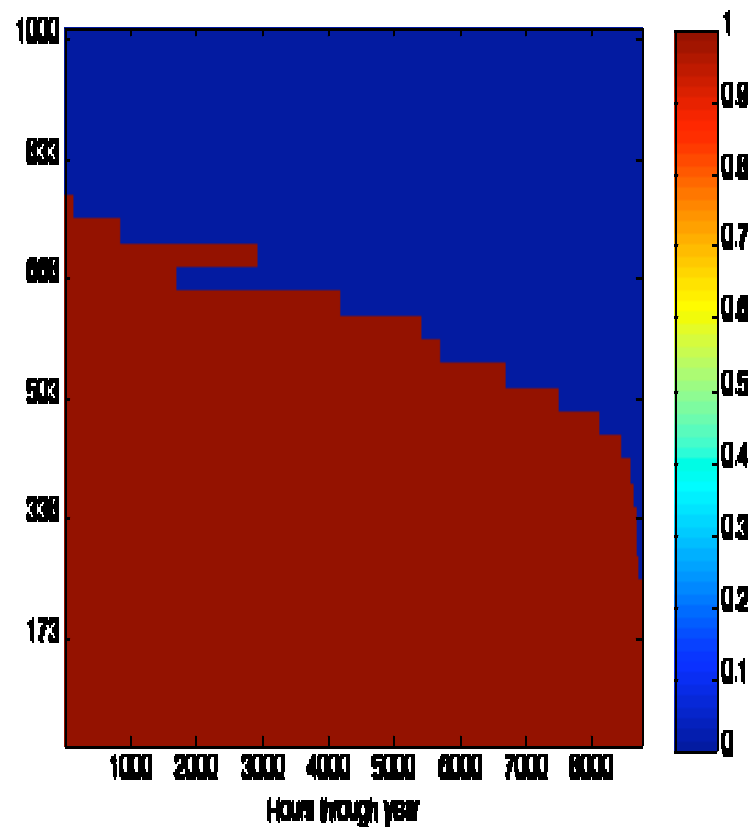
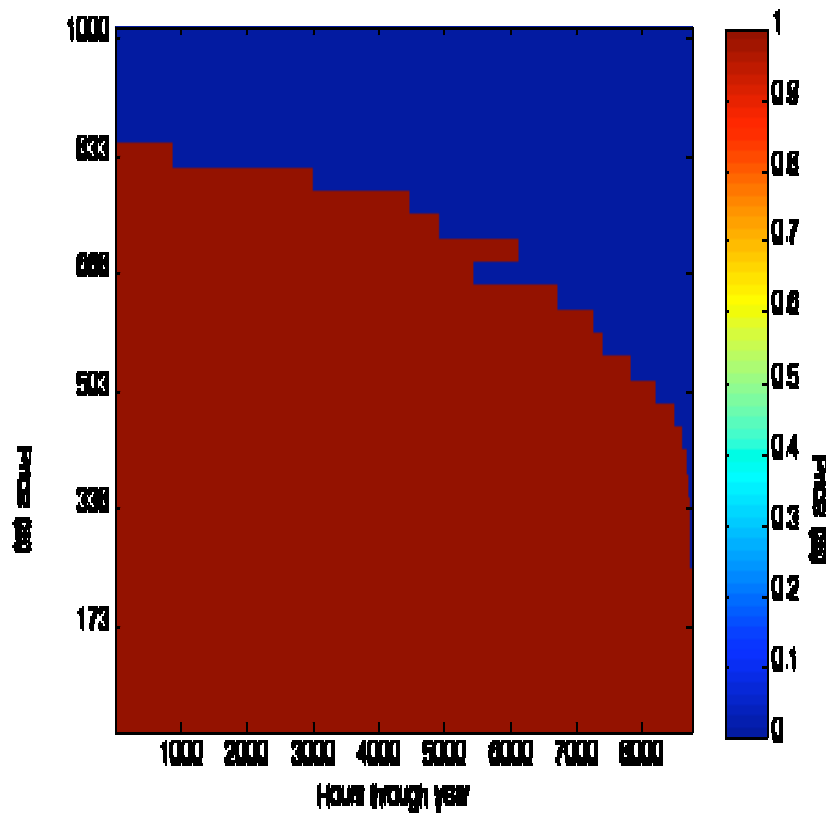
Value of the Curtailment Contract

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Results and Discussion



Curtailment call boundary when there are 40 hours (left) and 80 hours (right) of curtailment remaining



Curtailment call end boundary when there are 40 hours (left) and 80 hours (right) of curtailment remaining