Derivatives Pricing

AMSI Workshop, April 2007

Overview

Derivatives contracts on electricity are traded on the secondary market

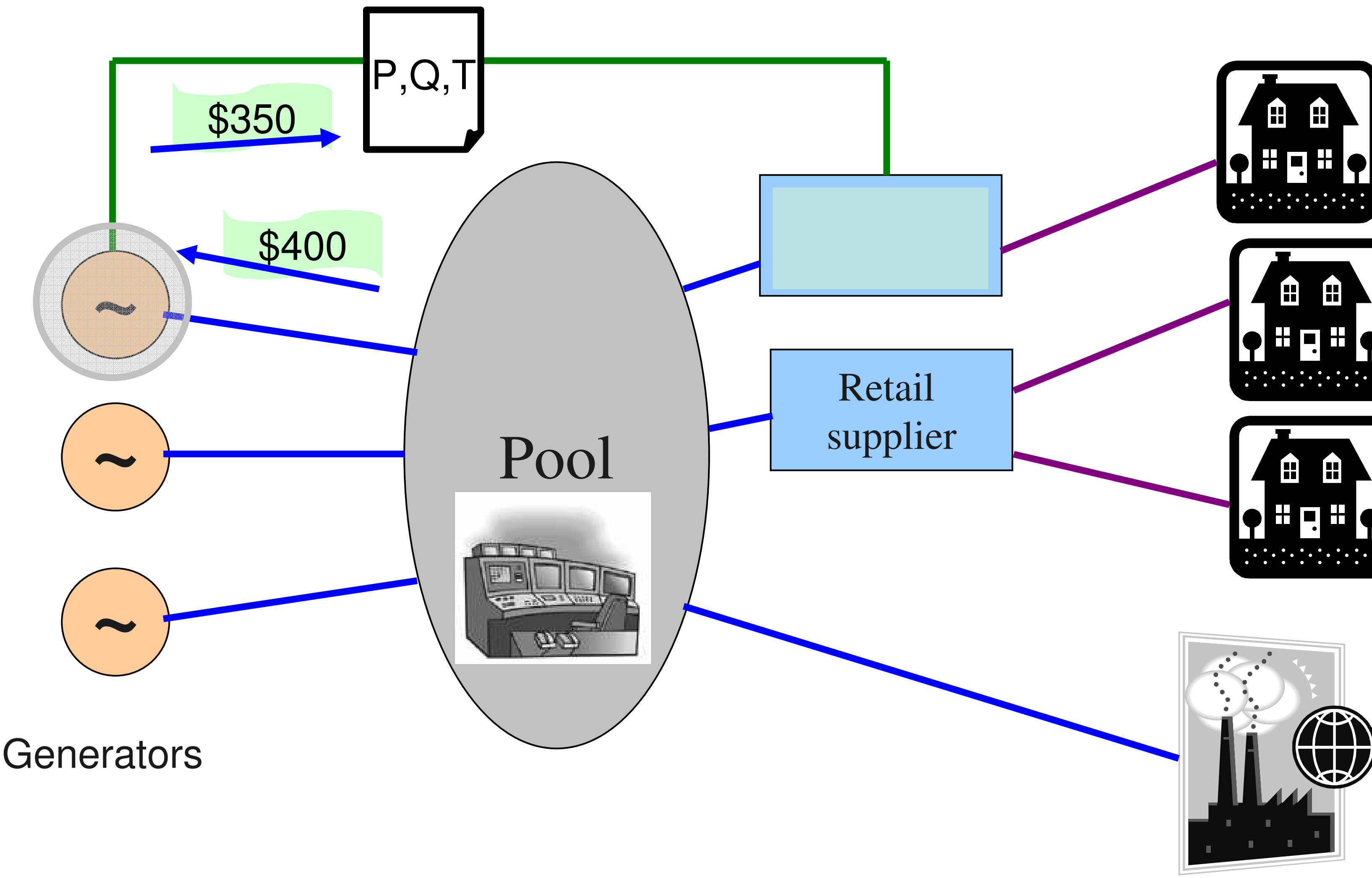
This seminar aims to:

Describe the various standard contracts available in the secondary

Expose some pragmatic pricing techniques for these derivatives

Structure

- Pool price dynamics
- Forward price dynamics
- Swaps
- Swaptions
- Asians
- Caps
- SRAs



P,Q,T
$350
$400
Pool
Retail supplier
Generators

Pool Price Models

- Stochastic differential equations
- Diffusion models:

$$dS = a(t,S)\,dt + b(t,S)\,dW$$

Standard financial model: GBM

$$dS = \mu S\,dt + \sigma S\,dW$$

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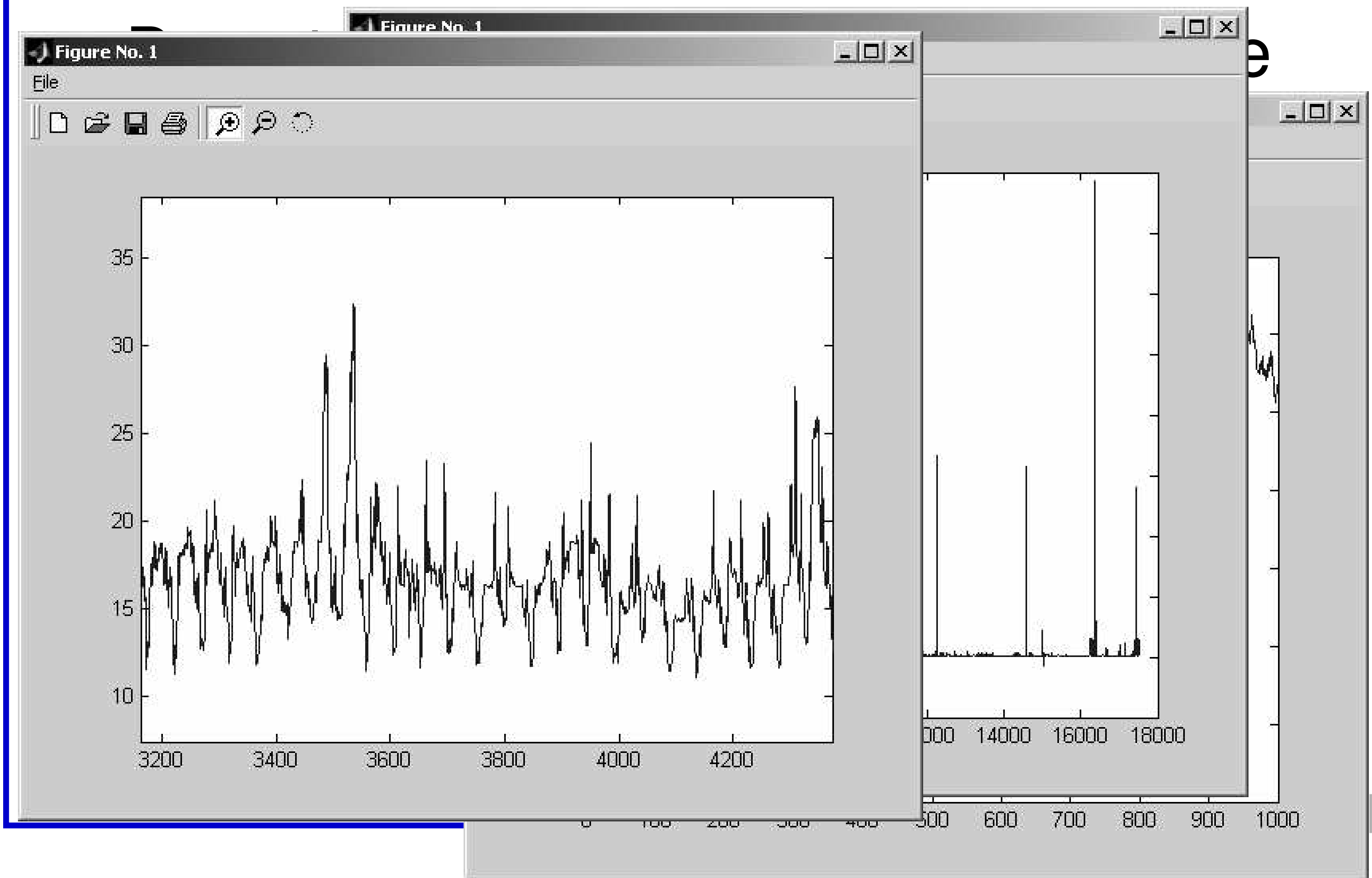
Diffusion models



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Jump-Diffusion models:

$$dS = a(t,S)\, dt + b(t,S)\, dW + K(t,S)\, dq$$

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MRJD: Standard model

- Mean reversion supported by physical market environment:
  - Plant flexibility
  - Load response
  - Opportunistic behaviour

$$dS = \alpha(\theta(t) - \ln S)S\,dt + \sigma(t)S\,dz + \kappa\,dq$$

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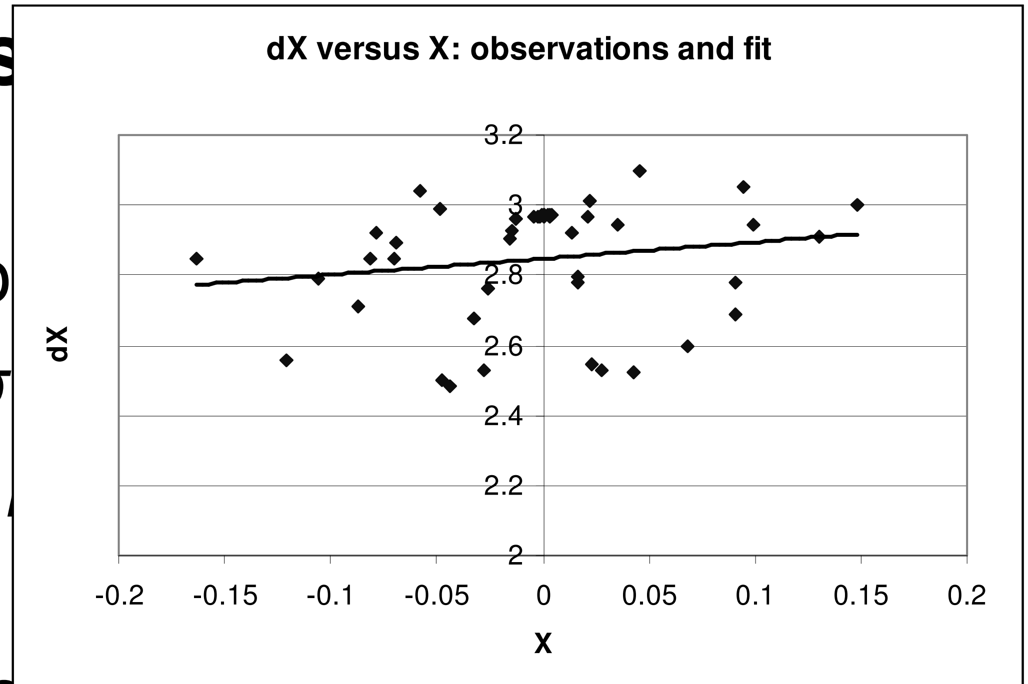
Estimating the parameters

- Step 1: Identify Jumps
- Method: **Recursive filter**
  1. Evaluate returns *dS/S*
  2. Find threshold of diffusic
  3. Identify outliers as jump
  4. Remove outliers from sa
  5. Return to 2
  - Continue until no new jumps found

S	dS/S			
19.38			STDEV	1.669453
18.08	-0.06708		3 x STDEV	5.00836
16.27	-0.10011			
15.85	-0.02581		COUNTIF > 5	4
16.12	0.017035			
16.38	0.016129			
15.01	-0.08364			
14.53	-0.03198			
12.87	-0.11425			
12.52	-0.02719			
11.98	-0.04313			

Estimating the parameters

- Step 2: Find volatility, drift and mean
- Method: **Regres**



1. *dS = α(μ-S)Sdt*
2. Apply Ito: *X* = lo

   dX = (*αμ* - 1/2 *σ*

Perform best fit by

y ≅ A + B X

3. B = α dt; αμ - 1/2 σ² dt = A; std(*y-y^*) = *σ √dt*

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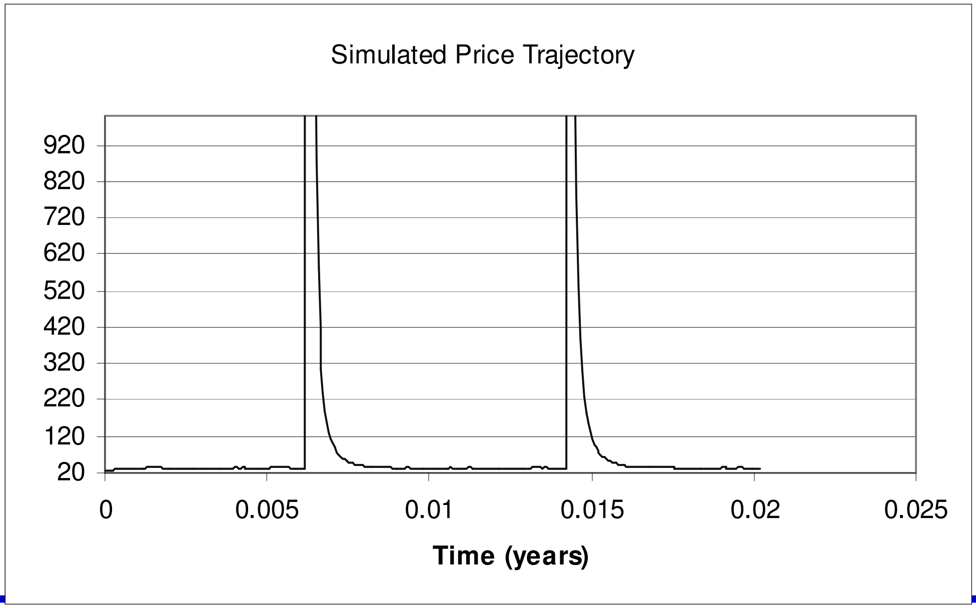
Simulating the process

$$dS = \alpha(\theta(t) - \ln S)S\,dt + \sigma(t)S dz + \kappa dq$$

- $S_{j+1} = S_j + \alpha(\theta(t_j) - \log(S_j)) \times S_j \times \Delta t + \sigma(t_j) \times Sj \times N\sqrt{\Delta t} + K \times \chi(\Delta t)$



Where N is normal and K is per the jump shape

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Swap derivative

- Forward contract for electricity
- Payoff of sold swap:

$$(K - P) \times V \times T$$

- Hold with physical generation:

$$P \times G \times T + (K - P) \times V \times T = KVT + P(G-V)T$$

- A purely financial cash-settled contract

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Swaps

- Q: What is fair price for entering into a forward contract (swap)?
- A: What the market is paying: the forward price.

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Observable Forward Prices

- In Australian secondary market:
  - Quarterly
  - Calendar
  - Peak, Offpeak, Flat
- Consistency relationships:
  - Flat = peak + offpeak
  - Calendar = Q1 + Q2 + Q3 + Q4

Peak = 7am – 10pm on working days

Offpeak is everything else

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Consistency Q to Cal

Period	Hours	Q Price $/MWh	Cal Price $/MWh	¬ Price $/MWh	Delivery (years)	Discount factor	Present Value ($/MWh)
Q1 2008	2184	86.00	51.61	34.45	0.125	0.992159	34.12
Q2 2008	2184	40.38	51.61	- 11.16	0.375	0.976661	- 10.96
Q3 2008	2208	40.15	51.61	- 11.39	0.625	0.961405	- 11.01
Q4 2008	2208	40.15	51.61	- 11.39	0.875	0.946388	- 10.84
Calendar 2008	8784	51.61					0.26

$250K on 100 MW deal

Comparability of Forwards and Futures

- Futures requires an initial margin
  - But earn interest at risk-free-rate
- Futures requires variation margins
  - But could equally be incoming or outgoing
- Futures can crystalise position now
  - But could be positive or negative
- Futures provide no credit risk
  - Can be significant

Swaps without visible prices

- What is fair price for
  - "Superpeak"
  - "Peak-end"
  - "Shoulderpeak"
  - "7-day peak"
  - Fully sculptured

Two approaches

- Method 1
  - Obtain profile of “expected” pool prices
  - Transform (scale) profile to fit liquid products

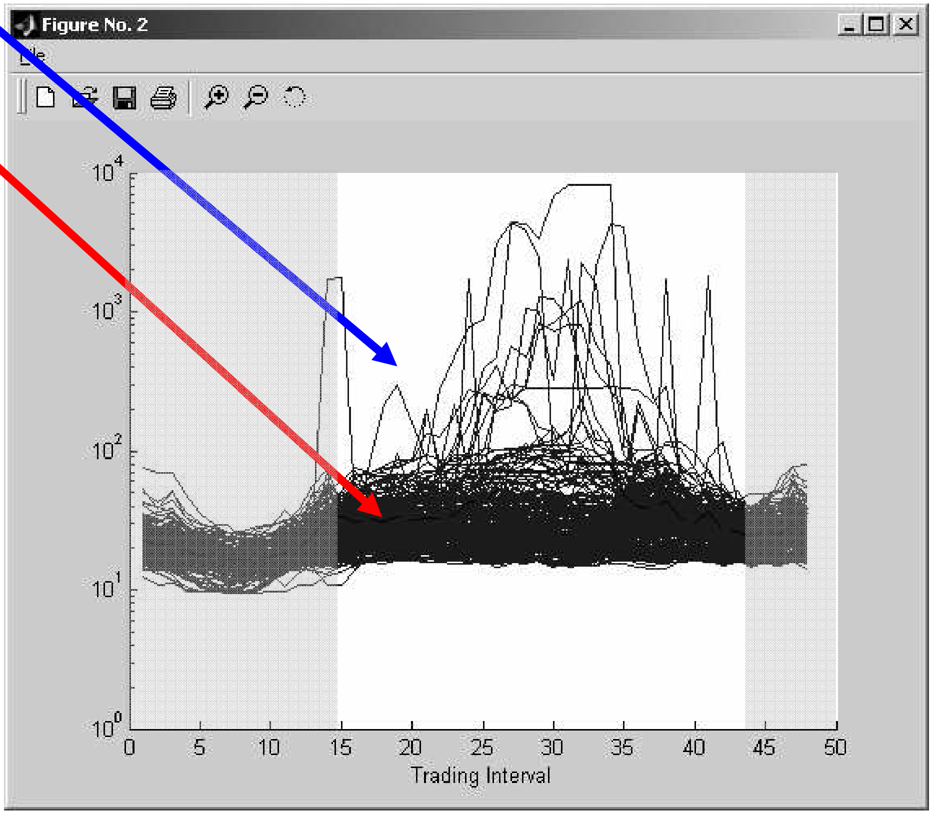
Method 2
  - Find relationship between historical pool outcomes
  - Apply relationship to liquid products

Method 1 example

- Display of 2004 working day pool prices
- Mean/median price shape
- Extract price over 15:44 for the 'profile'
- Rescale to fit observed peak forward price:

$$F_{halfhourly} = P_{profile} \times \overline{P}_{profile} / F_{peak}$$

Multiplicative transformation



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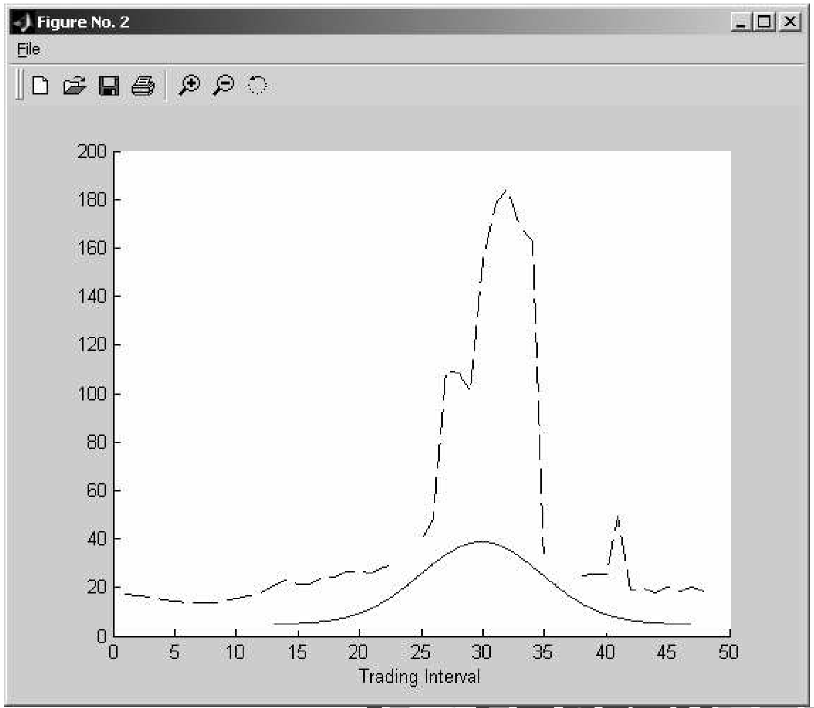
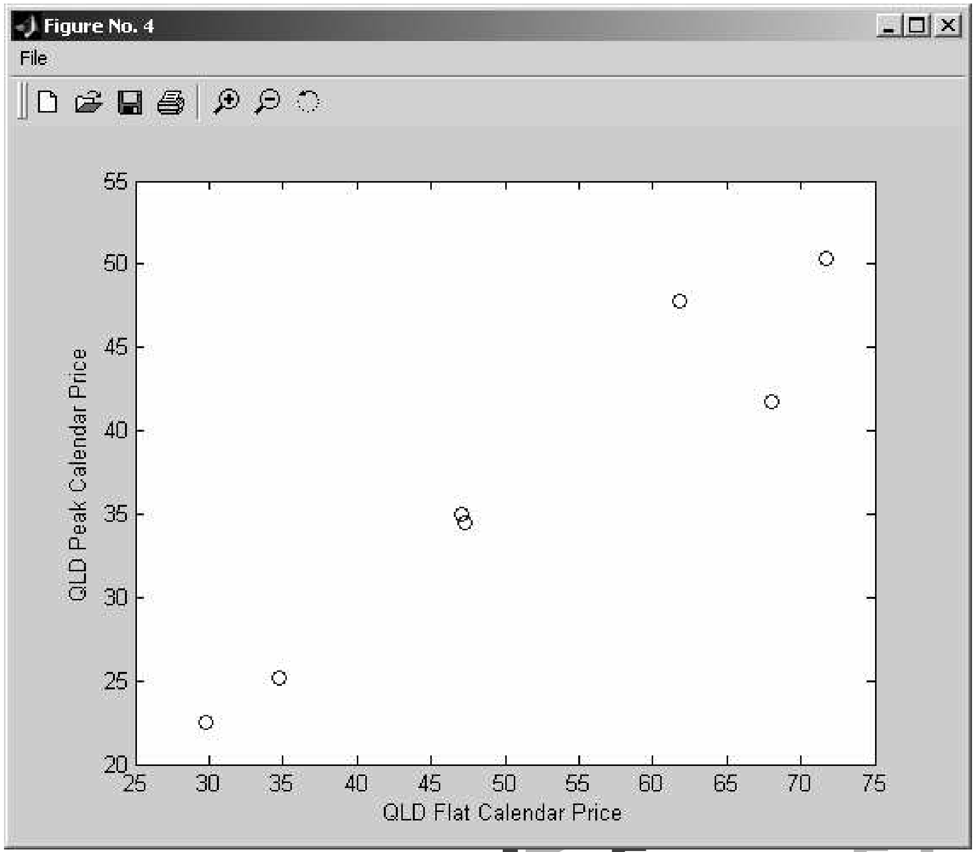
Kernel smoothing

- Historical outcomes are influenced by rare events
- Shown is Q1 2004 mean pool outcome
- Events at period 41 are not consistent
- The event might equally have occurred at period 40 or 42



Method 2 example

- Historical relationships are remarkably consistent
- Relationships other than best fit through the origin are justified



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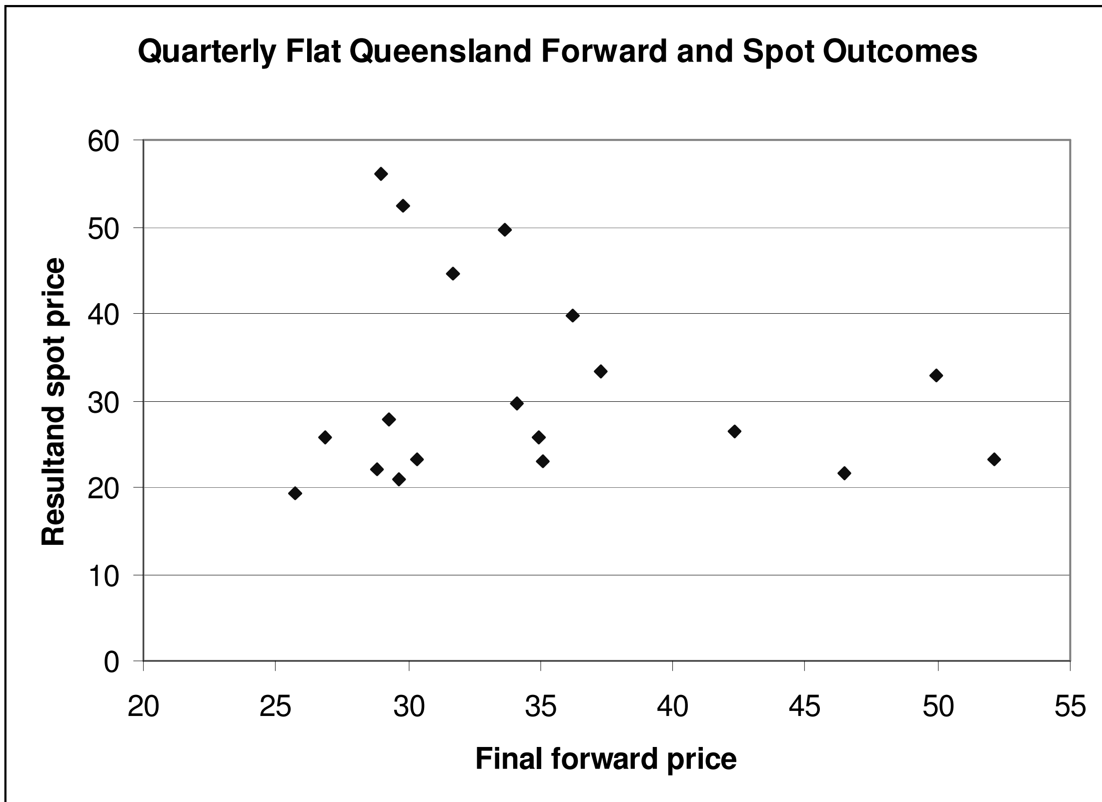
Fundamental query

- What is fundamental relationship between forward and spot prices?
- F = **E**(S)
- Two investigations:
  - What has happened historically?
  - Arguments in risk profiles

Historical Forward-Spot (flat)

- Anecdotally: “Forward price > Spot price”
- Empirically: Flat +$3.5 over data with stdev $15



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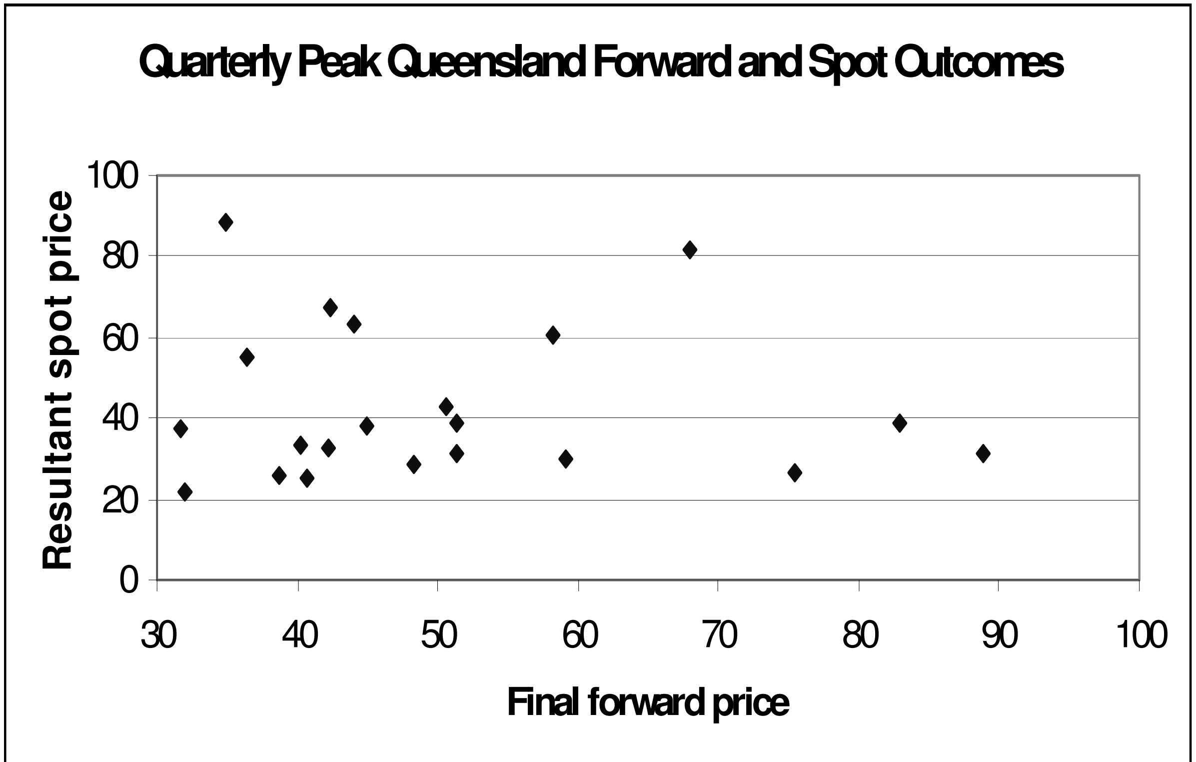
Historical Forward-Spot (peak)

- Empirically: Peak +$8 over data with stdev $27



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The “Risk Premium”

- The arguments:
- Pool price distributions are asymmetric
- Forward prices are reflective of spot price plus a risk premium
- “Risk premium” may be negative for periods of low pool volatility
- “Risk premium” is large for superpeak periods

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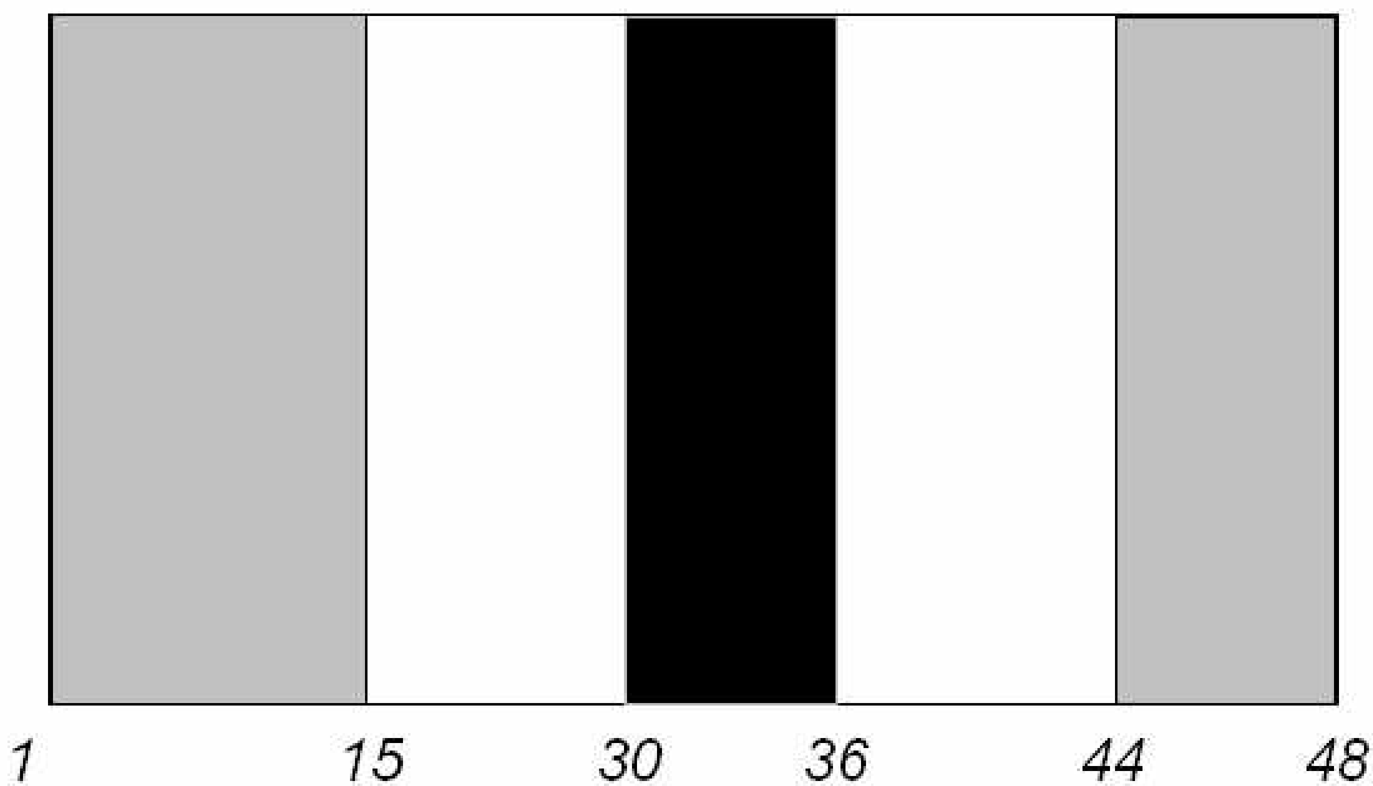
Risk Premium and Pricing



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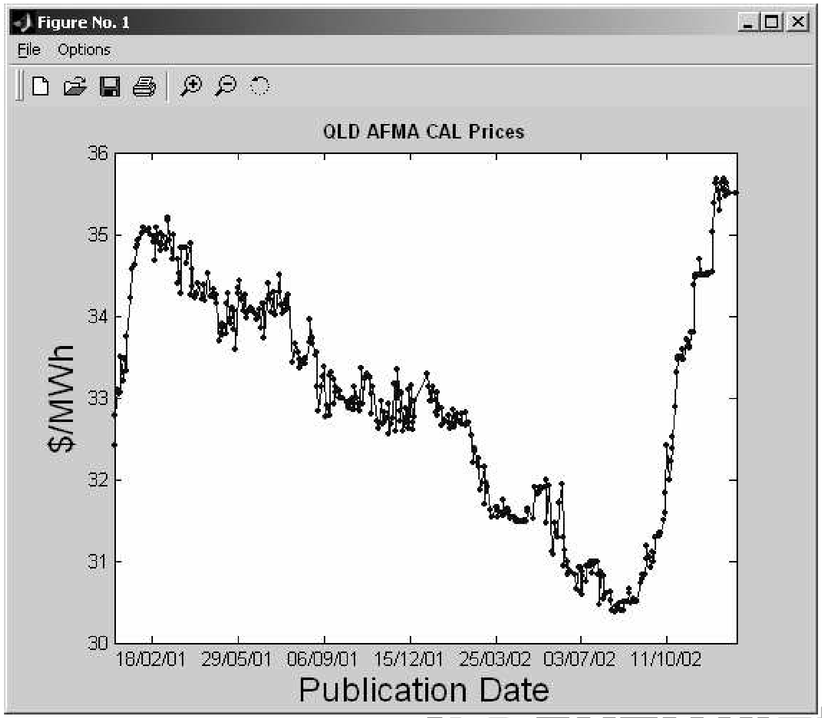
Swaptions

- An agreement between two parties that the option holder has the right to enter into a swap at a future date



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Option Pricing – Classical theory

- Price a call option on a stock (right to buy)
- 1973 – Black Scholes formula

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The Black-Scholes Equation

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0.$$

$$V(S,T) = \max(S - K, 0).$$

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The Black-Scholes Formula

$$C(S,T) = SN(d_1) - Ke^{-rT}N(d_2)$$

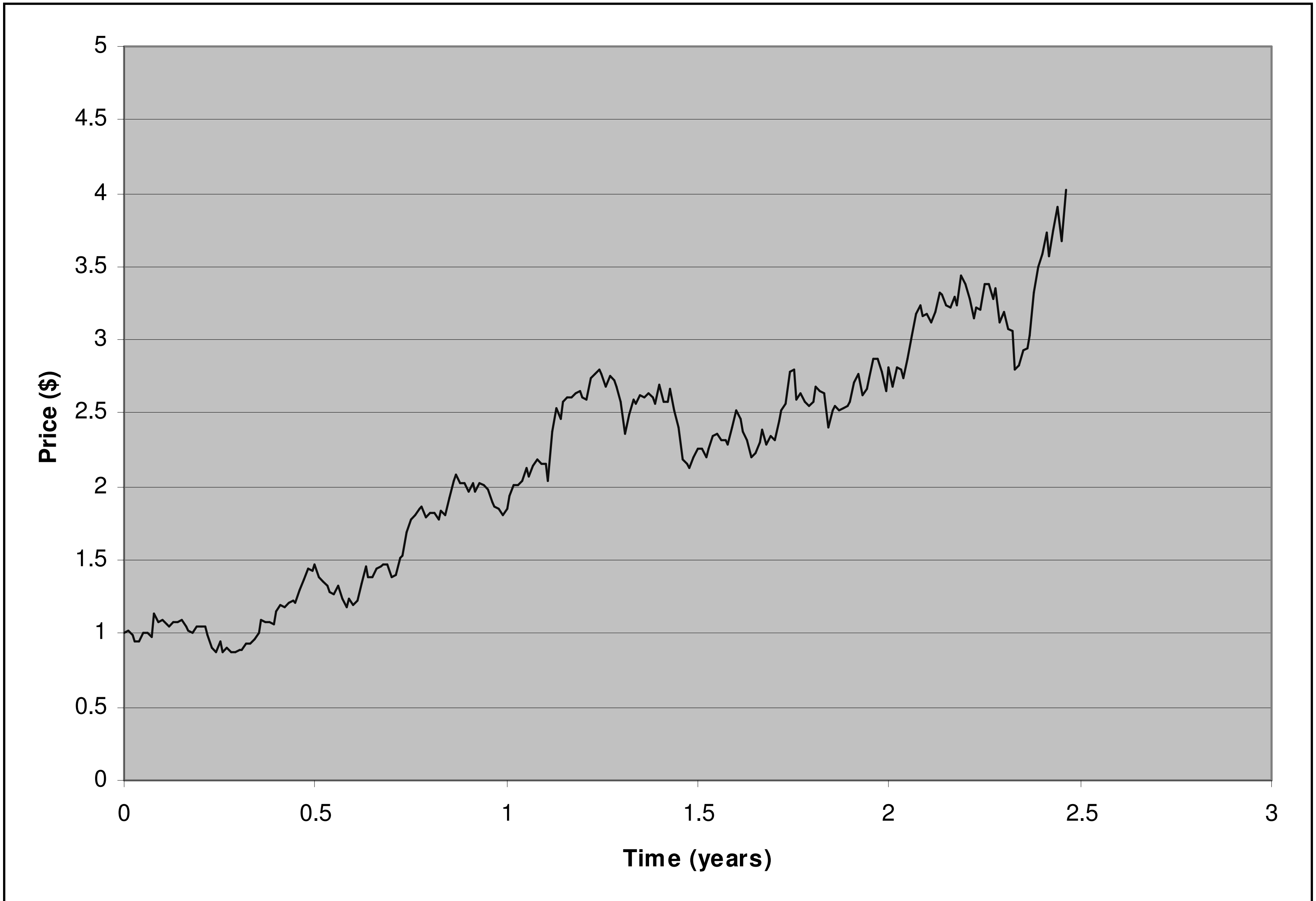
$$d_1 = \frac{\ln(S/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}.$$

Here *N* is the standard normal cumulative distribution function.

Black-Scholes Environment

- Stock price moves according to a random walk
- Random changes governed by annualised volatility σ
- Price increases biased by long-term annual growth μ
- Increments are uncorrelated
- Have access to lend/borrow at r



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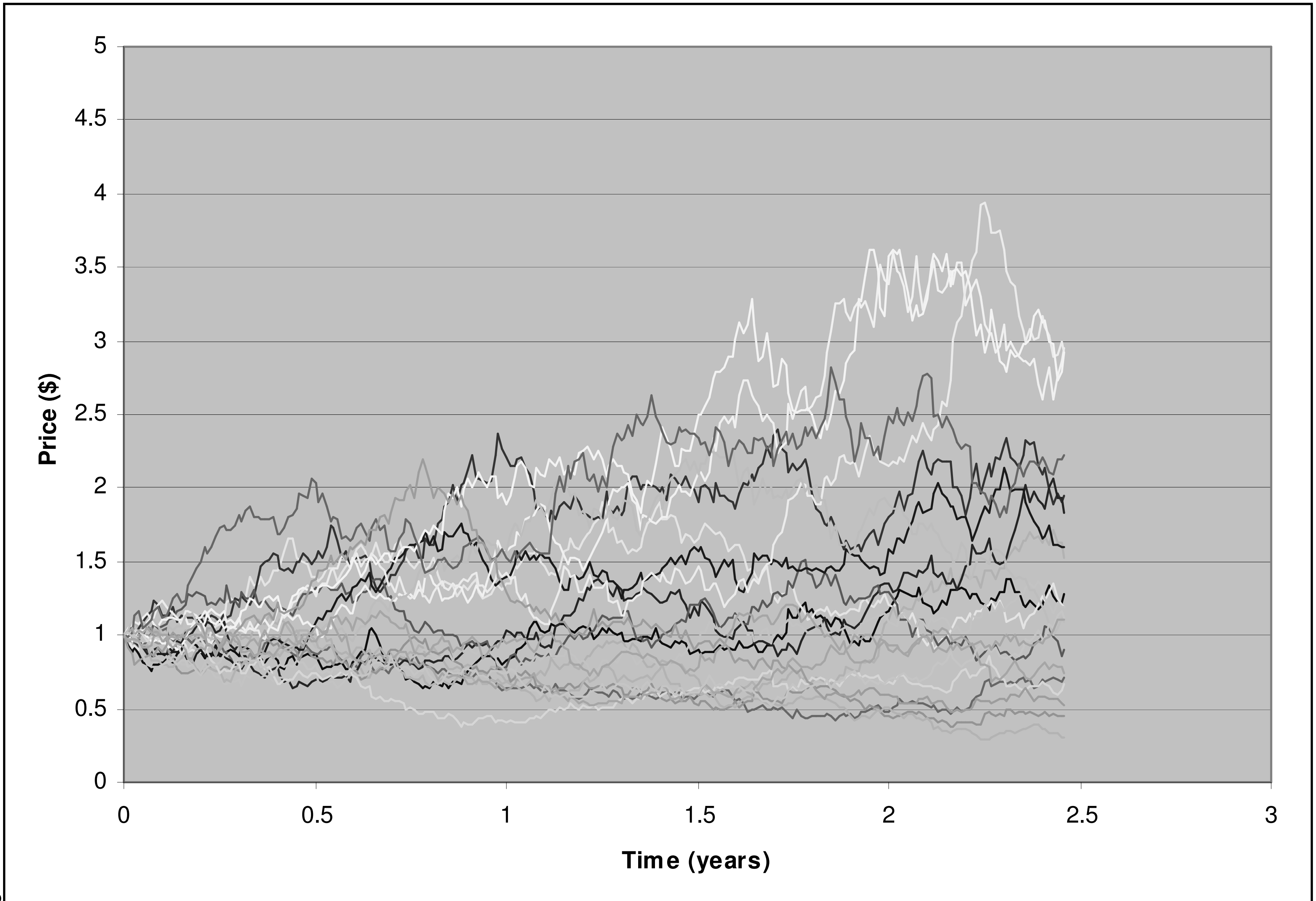
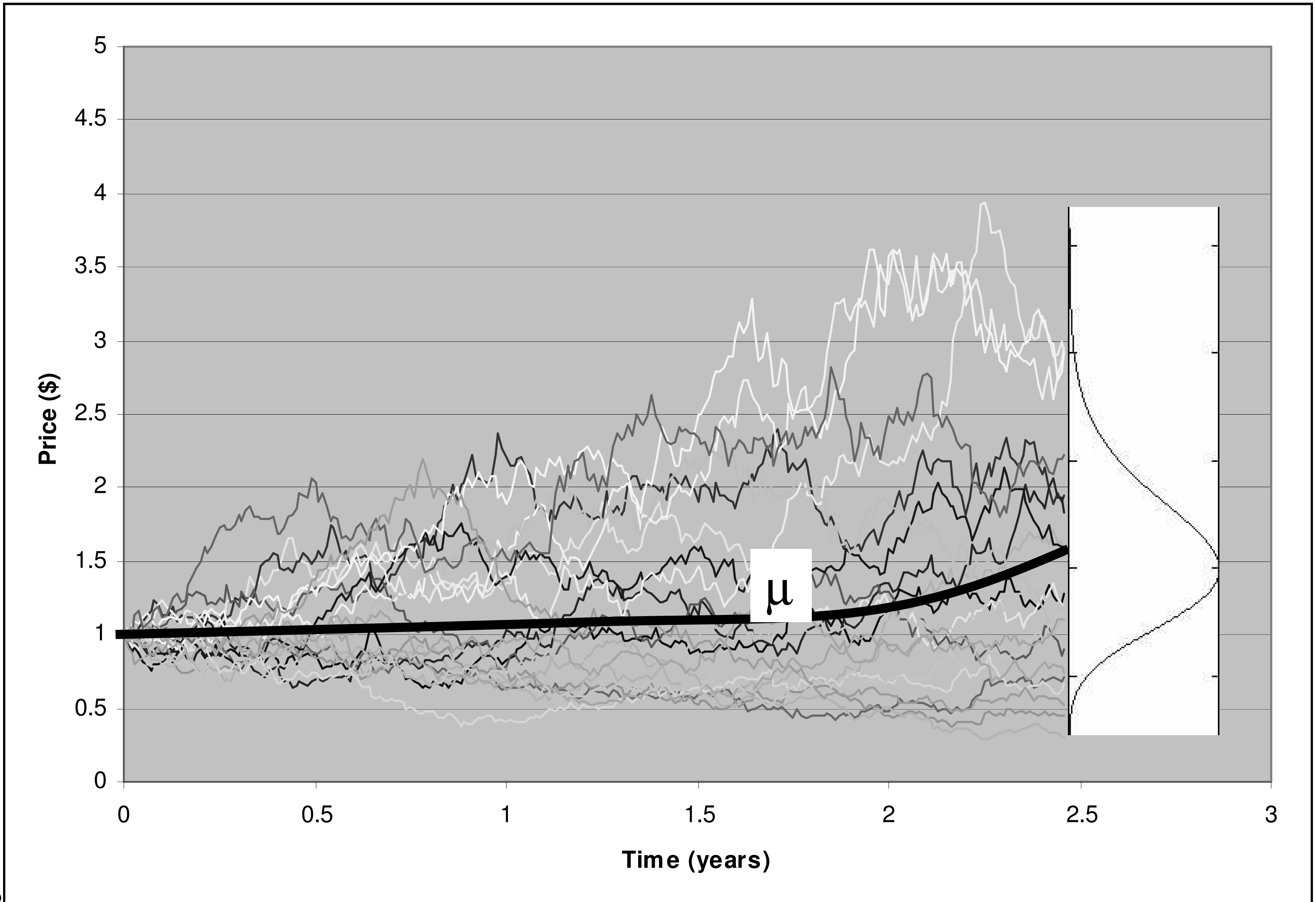


Price ($)
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
0
0.5
1
1.5
2
2.5
3
Time (years)



Price ($)
Time (years)
μ
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

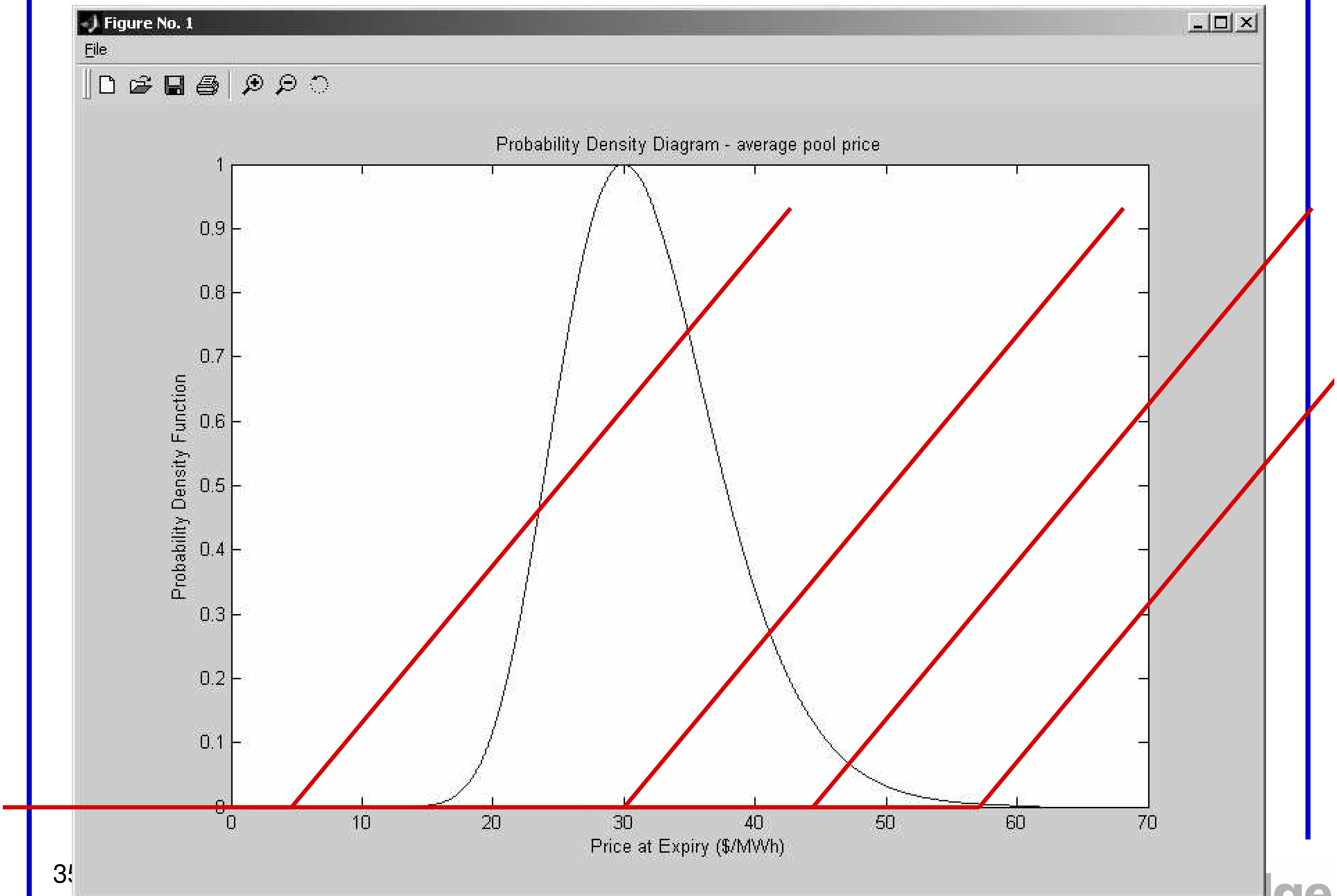


Figure No. 1
File
Probability Density Diagram - average pool price
Probability Density Function
Price at Expiry ($/MWh)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
10
20
30
40
50
60
70

Suggested pricing method

- Derive a lognormal probability distribution for the stock price using volatility σ
- Assume mean growth of μ
- Calculate expected (mean) payoff
- NPV at *r* to arrive at fair value today

Not quite right …

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0.$$

$$V(S,T) = \max(S-K,0).$$

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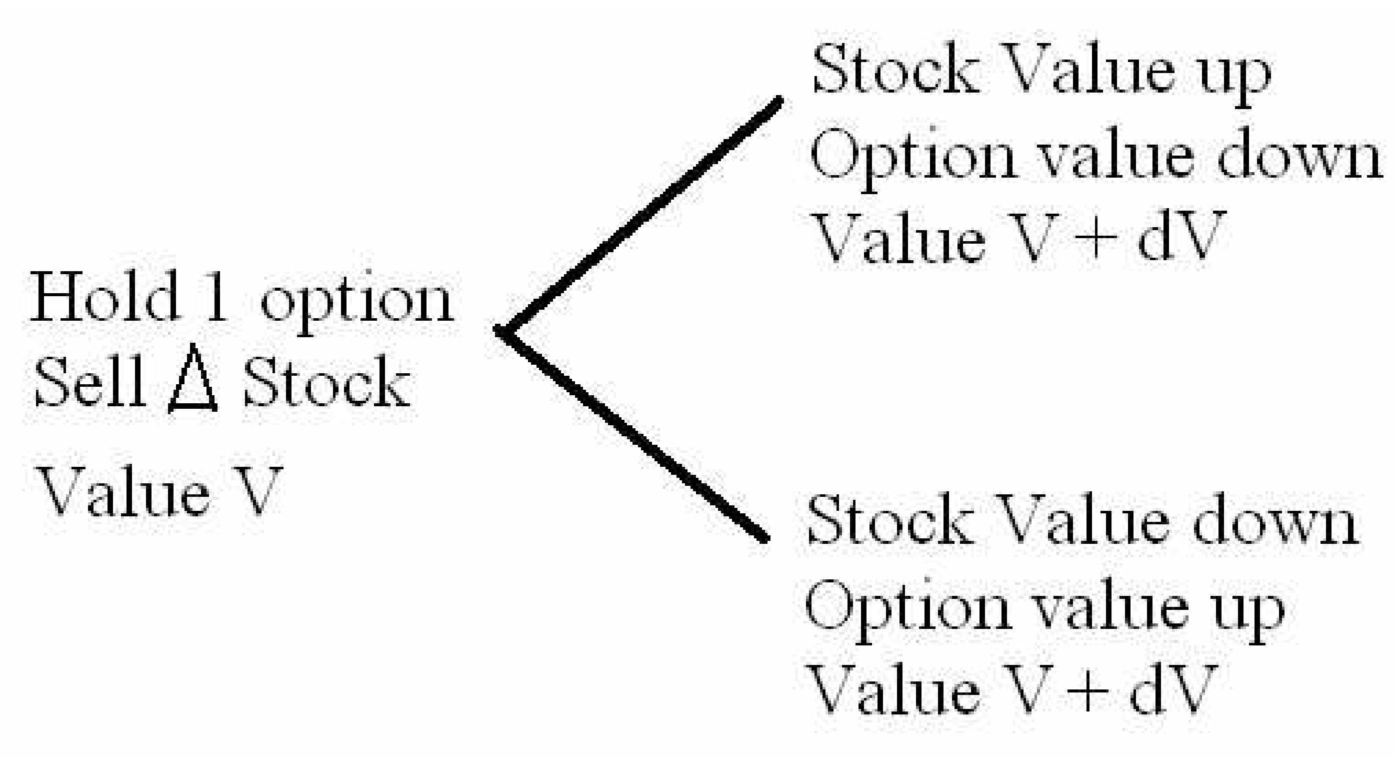
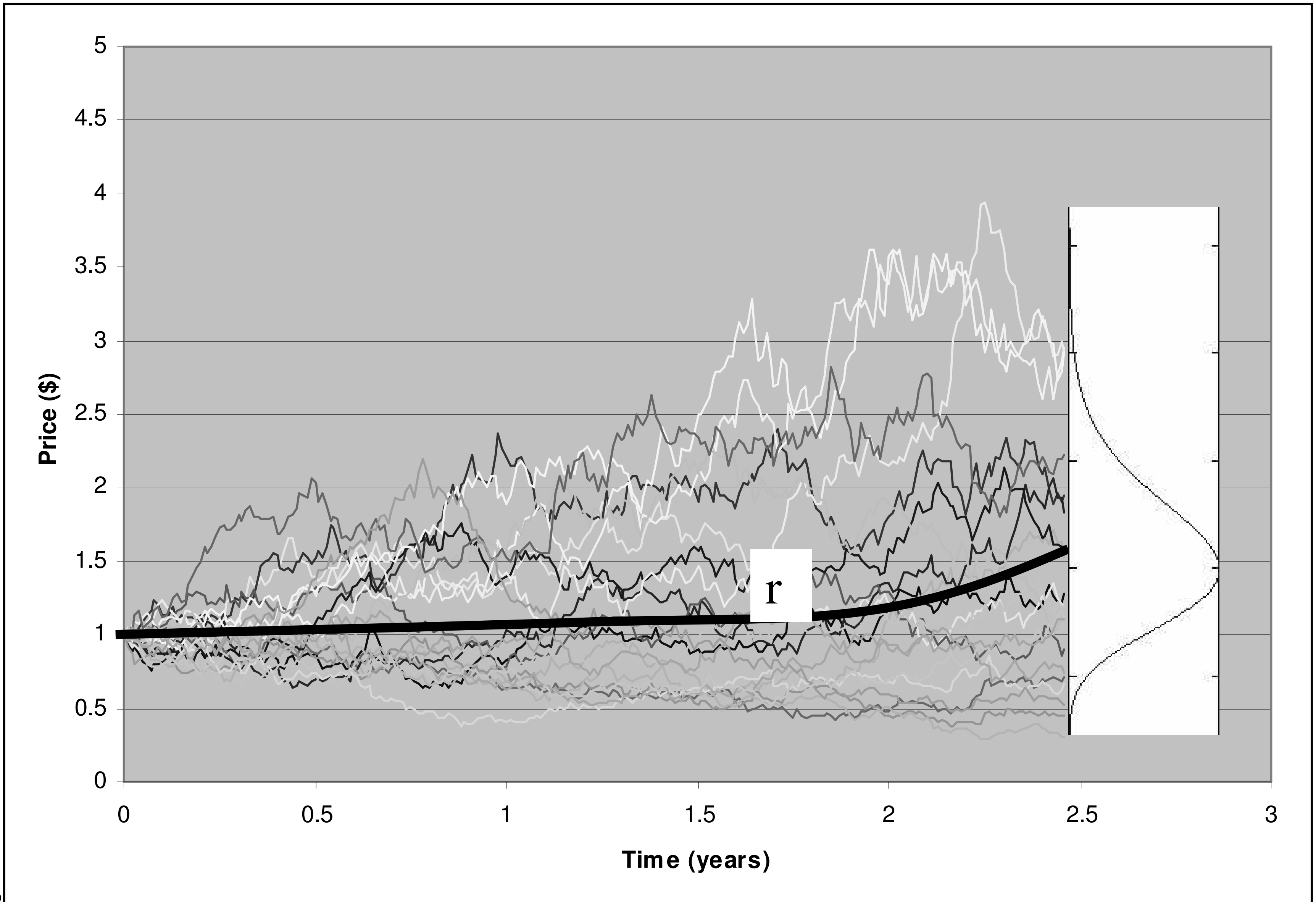
Delta Hedging





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Pricing Options on Swaps

- Hold a call swaption: right to buy a swap
- Swaption specifications:
  - Call/put, expiry date, strike price, cash/delivery
- Underlying specifications:
  - Prevailing forward price, volatility σ, growth μ, interest rate r

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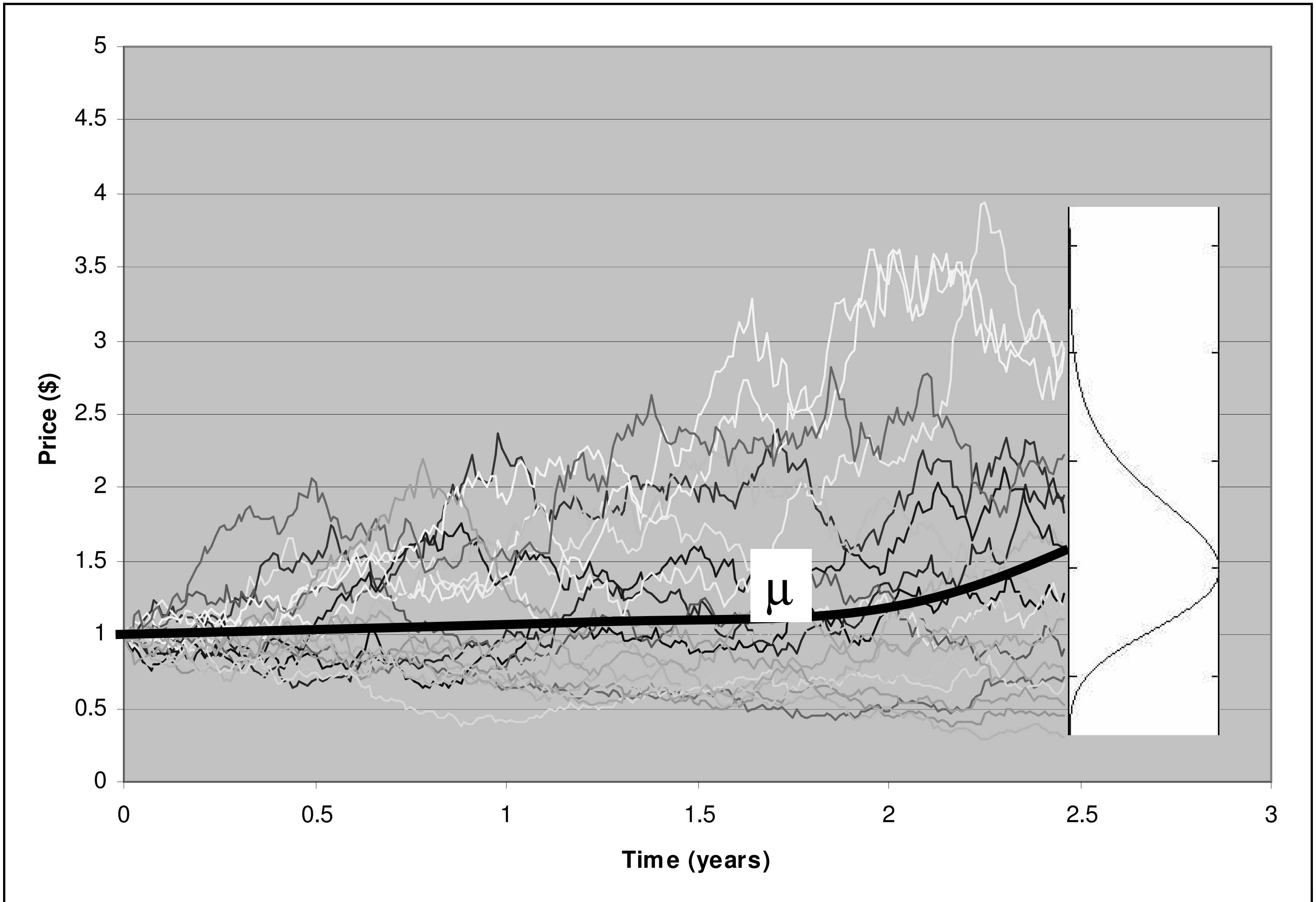
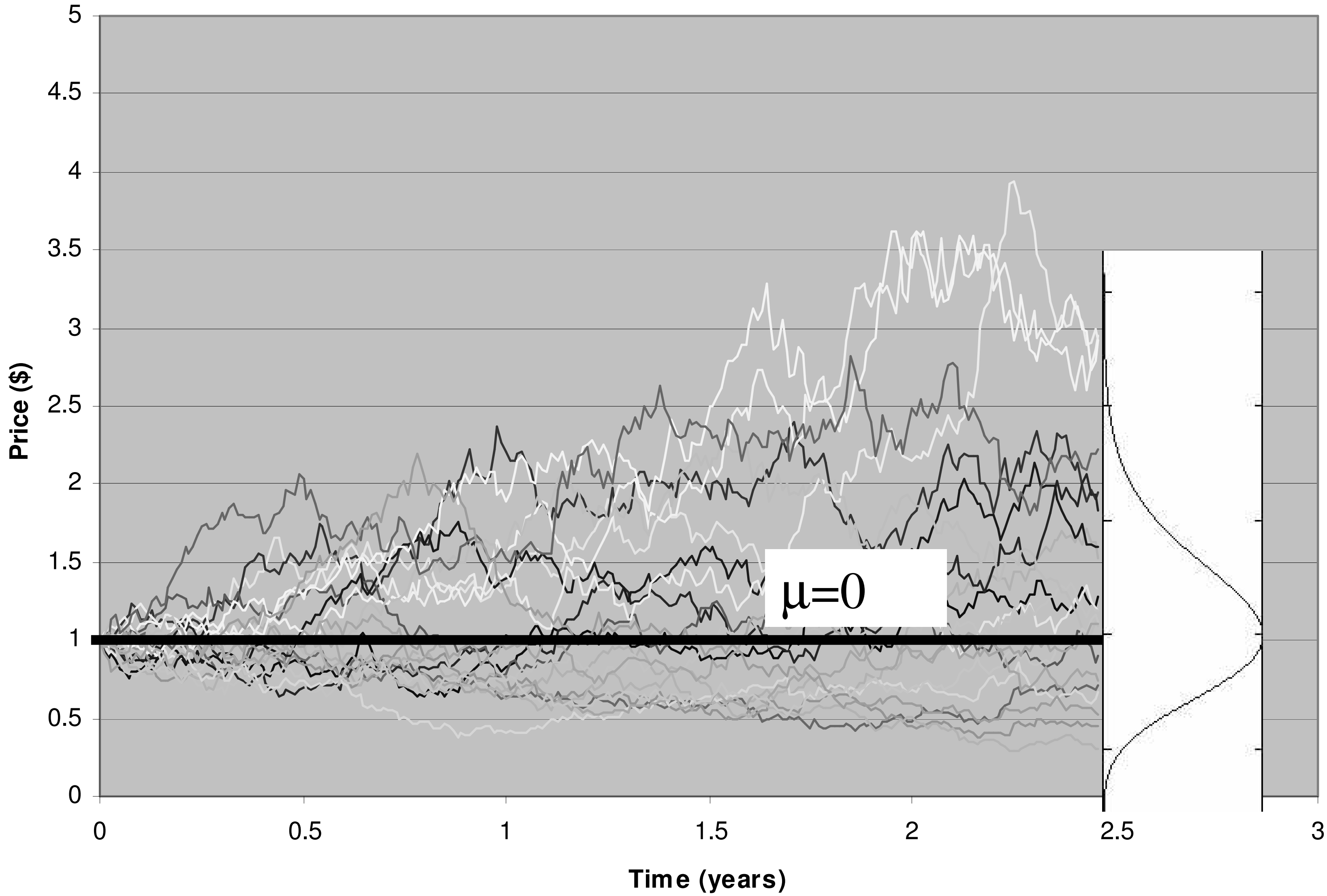


Price ($)
Time (years)
μ
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

Behaviour of forwards

- Forward price movements are unbiased ($\mu = 0$).
- With stocks, μ did not matter – delta hedged into *r*
- MAJOR DIFFERENCE: -
- A $35 stock is worth $35
- A $35 swap is worth $0
- The delta-hedged growth rate is zero



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Black's formula

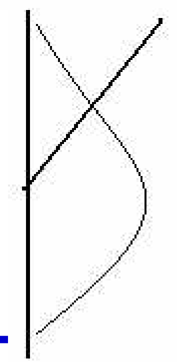
- Similar to Black-Scholes, but assumes zero growth on the underlying
- Closed form solution (analytic formula)

- Significant difference?
- $40/MWh call swaption at 15% volatility over 1.5 years
- BS = $4.79
- Black = $2.68

Cash flows and discounting

- Examine the cash flows of a swaption derivative:

$2.68 → $2.55



Transform from Black-Scholes

- Put r = 0; Multiply by exp(-$r \times t_{deliv}$)

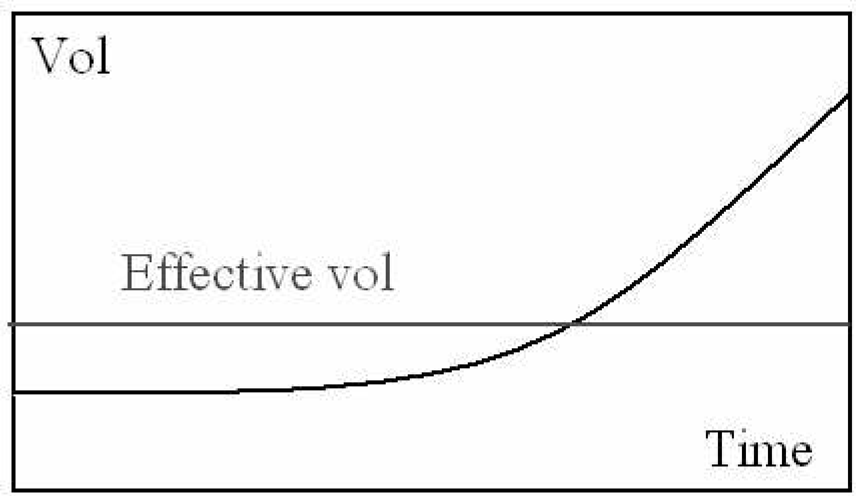
$$C(S,T) = SN(d_1) - Ke^{-rT}N(d_2)$$

$$d_1 = \frac{\ln(S/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}.$$

Nonconstant volatility

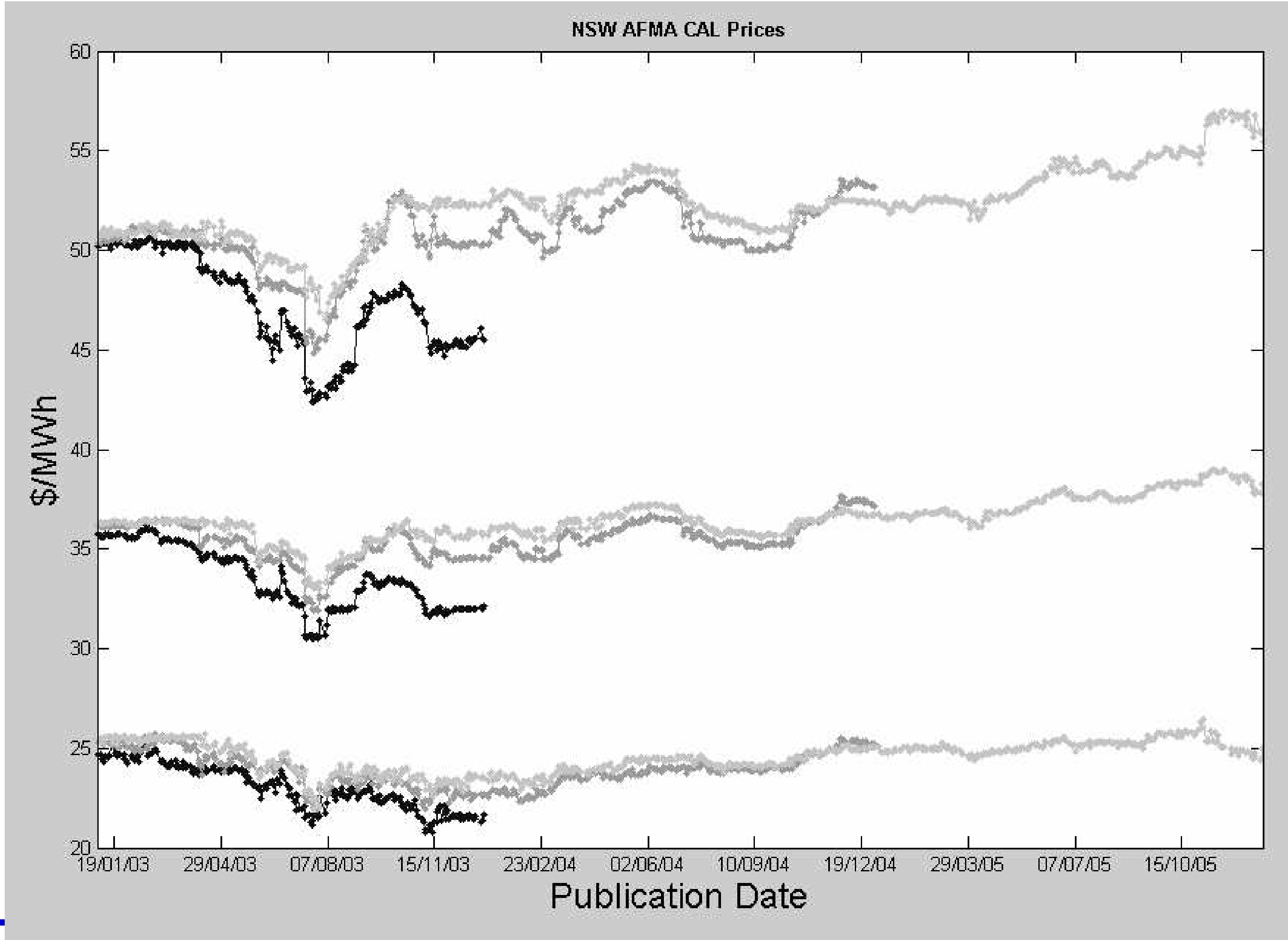
- As delivery approaches, volatility of the forward increases
  - More trading
  - More knowledge about physical
  - Samuelson effect
- $\sigma = \sigma_0 + \sigma_1 \exp(-\gamma(T-t))$
- Example: Cal 2004 flat



Strategies with swaptions

- Speculation
- Covered calls
- Hedging with a window of speculation
- Validity periods
- Large project deals

More on Forward Price Issues



Forward Price Issues

- Deterministic time-dependent volatility
- Stochastic volatility
- 250 day or 365 day
- Relation with spot
- Independence of daily increments
- Volatility estimation methods
- Unbiased movements
- Spread in the curve and relation to liquidity