Stochastic Models for Resource Markets

AMSI Workshop, July 2009
Energy Edge

- Energy Edge assists clients with financial risk management for exposures to commodities:
  - Electricity
  - Gas and coal
  - Carbon
  - Water

- Activities
  - Advisory Services
  - Quantitative Analysis
  - System Development
Electricity

• Optimal portfolio construction to hedge exposures to the National Electricity Market

• Pricing proposed or existing derivative contracts

• New power station viability studies
Gas and Coal

• Strategies for gas fired power stations to run with a constrained supply of gas

• Value of facilities to temporarily store gas

• Decision support for a (coal mine, generator) pairing to burn coal or sell it
Carbon

• How will the CPRS impact market prices for electricity

• Estimating the change in merit order for the generation fleet with a cost of carbon

• Constructing an optimal hedge for carbon risk mitigation under cash flow constraints
Water

• Contract management for purchase and sale of potable water in a regulated urban water network

• The impact of water restrictions on the electricity markets

• How to most profitably run a generator with a limited supply of cooling water
Point of commonality

• *Uncertainty*
  + Price risk
  + Volume risk
  + Counterparty risk
  + Regulatory risk
  + Behavioural risk
  + Weather risk

= Financial risk measures
Overview

• Three-part workshop
  – Part 1: Stochastic models
    • Describing and modelling the uncertainty
  – Part 2: Connection between electricity and water
    • Supply and demand influences
  – Part 3: Stochastic dynamic programming
    • Optimal management of a limited resource
Overview: Stochastic models

• Identify sources of uncertainty and illustrate tools for modelling
• Key topics:
  – Stochastic processes
  – Diffusion process
  – Poisson processes
  – Markov chains

Description, solution, simulation

For modelling continuously variable quantities, like consumer demand

For modelling shock events, like price spikes and plant failure

For more general modelling of time-varying phenomenon, like electricity spot price
Stochastic process

- A family of random variables $X_t$, where $t$ is a parameter running over an index set (time).
- Distinguished by their state space (range of $X$), index set $T$ and dependencies between $X_t$.
- Continuous processes
  - Continuous $\{t\}$
- Discrete processes
  - Discrete $\{t\}$
Electricity demand

- Electricity consumption levels pose a risk for market players:
  - Generation production levels to meet demand
  - Retailers’ exposures on consumption levels
- Demand is a continuously varying quantity
  - Amenable to modelling by a *stochastic differential equation* to describe a *diffusion process*.
- What are the characteristics to capture?
The National Electricity Market

• To put in context, we give a description of the wholesale electricity market (NEM)
Place of the Electricity Market

- Weather markets
  - Water market, weather derivatives
- Suppliers
- Electricity markets
  - Financial
  - Physical
- Consumers
- Commodity markets
  - Coal market
  - Gas market
  - Carbon market
  - Environmental markets

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Physical Infrastructure

-6% internal

-9% transmission

-6% distribution

Demand is measured here

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NEM

• The “pool” market
• Wholesale market for electricity
• Divided into half-hourly trading intervals
  – 48 observations per day for price, demand
• Demand is always met by supply in real time
• Prices are continually reset according to supply/demand balance
Demand Modeling

• The characteristics of consumer demand
• Time series of demand data for, say, QLD
• Relatively exogenous
• Limited demand side response
• Aggregation of different user types:
  – Industrial
  – Commercial
  – Domestic
Large time scale
Monthly average demand

![Graph showing monthly average demand over time]
Randomness over the range
Daily maximum demand
Daily Variation

10 Jan 2008 to 10 Feb 2008

10 Jun 2008 to 10 Jul 2008
Systematic diurnal pattern
Stochastic differential equation

\[ dS = a(t,S) \, dt + b(t,S) \, dW \]

That is,

• Over time step \( dt \), the price \( S \) moves by a small amount \( dS \) which is a random amount \( b(t,S) \, dW \). Here \( b \) is a volatility parameter.

• For modelling, \( dW \) is a normally distributed random variable.

\[ S(\text{next}) = S(\text{current}) + a \times dt + b \times \sqrt{dt} \times N(0,1) \]

• We can generate a normal random variable by:

\[ N(0,1) = \text{normsinv}(\text{rand}()) \]
Development of the SDE

\[ dS = a(t,S) \, dt + b(t,S) \, dW \]

Let \( W_t \) be a Brownian motion path.

Omit the references to probability.

What is a Brownian motion:

- Continuous,
- Nowhere smooth,
- Unbounded variation
Brownian motion

• $W_t$ is a Brownian motion if:
  – **Gaussian**: for each $s$ and $t$, $W_{t+s} - W_s$ has a normal distribution, mean 0 and standard deviation $\sqrt{t}$
  – **Independent increments**: for each $0 \leq t_0 \leq t_1 \leq \ldots \leq t_n \{W_{t_j} - W_{t_{j-1}}\}$ are independent
  – **Continuity**: $W_t$ is a continuous function
Sample trajectories
Simulation of a solution

\[ dS = a(t,S) \, dt + b(t,S) \, dW \]

Euler method:

\[ S(t+1) = S(t) + a(t,S(t)) \times dt + b(t,S(t)) \times dW \]
Relation to demand

\[ dS = dW, \ S(0) = 5,100 \]

\[ dS = 200 \times dW, \ S(0) = 5,100 \]
Enforce the shape

Mean reverting SDE
\[ dS = \theta (\mu - S) \, dt + b \, dW \]

Here, \( \theta \) is the mean reversion rate
\( \mu \) is the mean reversion level

Termed Ornstein–Uhlenbeck process.
OU Process

![Graph showing OU Process with three lines labeled S1, S2, and S3. The graph ranges from 0 to 8 on the x-axis and from 3500 to 7500 on the y-axis. The lines represent different scenarios or states over time.]
Impose daily shapes

Mean reverting SDE

\[ dS = \theta (\mu - S) \, dt + b \, dW \]

Implement the mean reversion level as the mean daily shape for demand

\[ dS = \theta (S_{\text{mean}} - S) \, dt + b \, dW \]
Reversion Level = Mean Daily Shape

\[ \theta = 10, \quad b = 1000 \]
Ito’s lemma

• Mechanism for describing a new random process which is derived from another

• Primary process:
  \[ dS = a(t,S) \, dt + b(t,S) \, dW \]

• Secondary process: \( V(t,S) \)
  \[
  dV = \frac{\partial V}{\partial t} \, dt + \frac{\partial V}{\partial S} \, dS + \frac{1}{2} b^2 \frac{\partial^2 V}{\partial S^2} \, dt
  \]
Solution to the OU

dS = \theta (\mu - S) \, dt + b \, dW

Put f = S \exp(\theta t);
Then df = \partial f/\partial t \, dt + \partial f/\partial S \, dS
+ \frac{1}{2} \partial^2 f/\partial S^2 \, dS

df = \theta S \exp(\theta t) \, dt + \exp(\theta t) \, dS

\[ df = \theta S \exp(\theta t) \, dt + \exp(\theta t) \times [ \theta (\mu - S) \, dt + b \, dW ] \]

\[ df = \theta \mu \exp(\theta t) \, dt + \exp(\theta t) \, b \, dW \]
Solution to the OU ctd

\[ df = \theta \mu S \exp(\theta t) \, dt + \exp(\theta t) \, b \, dW \]

\[ \int_0^t df = f(t) - f(0) \]

\[ = \int_0^t \theta \mu \exp(\theta t) \, dt + \exp(\theta t) \, b \, dW \]

\[ = \int_0^t \theta \mu \exp(\theta t) \, dt + \int_0^t \exp(\theta t) \, b \, dW \]

\[ f(t) = f(0) + \int_0^t \theta \mu \exp(\theta t) \, dt \]

\[ + \int_0^t \exp(\theta t) \, b \, dW \]

\[ f = S \exp(\theta t); \]
Solution to the OU

\[ S(t) \exp(\theta t) = S(0) \]

\[ + \int_0^t \theta \mu \exp(\theta t) \, dt + \int_0^t \exp(\theta t) \, b \, dW \]

We were comparing the mean outcomes. Do this again.

\[ \mathbb{E}(\int_0^t \exp(\theta t) \, b \, dW) = 0 \]

\[ \mathbb{E}(S(t)) = S(0) \exp(-\theta t) \]

\[ + \exp(-\theta t) \int_0^t \theta \mu_t \exp(\theta t) \, dt \]

Now, if I want to have my simulated mean agree with the observed mean:
Calibrating the OU

\[ E(S(t)) = \exp(-\theta t) \left[ S(0) + \int_0^t \theta \mu_t \exp(\theta t) \, dt \right] \]

\[ S_{\text{mean}} = \exp(-\theta t) \left[ S(0) + \int_0^t \theta \mu_t \exp(\theta t) \, dt \right] \]

\[ S_{\text{mean}} \exp(\theta t) - S(0) = \int_0^t \theta \mu_t \exp(\theta t) \, dt \]

Differentiate:

\[ \mu_t = \left( \frac{dS_{\text{mean}}}{dt} - \theta S_{\text{mean}} \right) / \theta \]
Sample trajectory
Mean reversion level

\[ \text{MEAN} = \frac{(dS_m/dt + \theta (S_m - S_0))}{\theta} \]
Multiple regions

- The demand is observed concurrently in multiple regions
- Strong correlation exists
- Accurate risk measurement requires incorporation of correlation
Correlation Illustrated
Inducing the correlation

\[ dS_Q = a(t,S_Q) \, dt + b(t,S_Q) \, dW_Q \]
\[ dS_N = a(t,S_N) \, dt + b(t,S_N) \, dW_N \]

We will make the random movements correlated:

\[ dW_Q = \text{random draw} \]
\[ W^* = \text{random draw (intermediate step)} \]
\[ dW_N = \rho \, dW_Q + \sqrt{1-\rho^2} \, W^* \]
More complex correlation

![Graph showing complex correlations between nsw, qld, vic, sa]
Process

• Correlation array, volatility vector
• *Clean* the correlation array
  – (want a positive definite covariance matrix)

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Correlation QLD-NSW = 0.9, Correlation NSW-VIC = 0.9,
Correlation QLD-VIC = 0
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• Cholesky decomposition (L)
  – Correl = $L L^T$
• $L$ * uncorrelated random numbers
  ➔ correlated random numbers